# General Relativity (PHYS 325) PS 6

Spring 2026 v1.5

- A freely-falling Calvin

This week we have a little Christoffel computing, work with geodesics on a sphere, and more on accelerating observers. It looks like we are on track to assemble Einstein's equation before break. After break we will study the main current applications: Gravitational waves, black holes, and cosmology.

**Reading:**

- We discussed $T^{\mu\nu}$ in Schutz Chapter 4 pages 84-89, sections 4.2 and 4.6 and page 100. We skipped the rest.
- Schutz Chapter 5 sections 1 - 4 (We skip section 5.5.)
- A look ahead - Schutz Chapter 6 on Riemannian geometry

**Problems: Solutions due by 11 PM on Thursday, March 5**

All numbered problems are from Schutz.

(1) "Oops!" What is wrong with the following equation?

$$\frac{dp^\mu}{d\tau} = qF^{\mu\nu}U^\beta$$

Using the metric $g_{\alpha\beta}$ how might you correct the typo?

(2) GPS satellites emit signals at a constant rate, as measured by an onboard clock. They orbit at an altitude of $20.2 \times 10^6$ m.

(a) Using the approximate relativistic correction you found in PS 1 problem 7 find the relative fractional difference

$$\frac{\Delta t - \Delta \bar{t}}{\Delta t}$$

in the clock rates onboard the GPS satellites and on Earth. GPS satellites travel at $v_s = \sqrt{GM/r_s}$, where $r_s$ is the radius of the satellite's orbit. The speed of the surface of Earth in the (approximate) inertial frame at Earth's center is 0.46 km/s.

(b) Using the effect of gravity on clock rates we derived in class, find the fractional difference $(\Delta\tau_B - \Delta\tau_A)/\Delta\tau_A$ due to the gravitational potential.

(c) Find the net effect. Comment on the results.

(3) 5.2 On tides

(4) 5.7 Computing the transformation as we did in class

(5) 5.8 Computing with 1-forms and vectors in 2D

(6) DELAYED TO NEXT WEEK 5.21 ( 2 pts.) The last of the "Rindler spacetime set" of 2.19, 2.21, and 5.21.

(7) Consider the 2D spacetime metric with coordinates $(t, r)$,

$$ds^2 = -\left(1 - \frac{1}{\sqrt{r^2 + a^2}}\right) dt^2 + \left(1 - \frac{1}{\sqrt{r^2 + a^2}}\right)^{-1} dr^2,$$

where $a$ is a constant.

(a) Write the metric and its inverse as $2 \times 2$ arrays in these coordinates.

(b) Compute the non-vanishing Christoffel symbols. Please show your work.

(8) DELAYED TO NEXT WEEK Compute the Christoffel symbols $\Gamma^t_{tt}$ and $\Gamma^i_{tt}$ for the metric shown in equation (7.8) assuming that the function $\phi = \phi(t, x, y, z)$ is a function of all the coordinates.