General Relativity (PHYS 325) PS 7

Spring 2026 v1.0

This week we extend our skills with the covariant derivative or connection, start computing the Riemann tensor. A few of the problems are practice computing Christoffel symbols.

Reading:

Schutz Chapter 6 sections 3 - 7 (Feel free to read the first two sections for general interest.)

Problems:

All numbered problems are from Schutz.

(1) 5.21 (2 pts.) The last of the "Rindler spacetime set" of 2.19, 2.21, and 5.21.

(2) Compute the Christoffel symbols Γ^t_{tt} and Γ^i_{tt} for the metric shown in equation (7.8) assuming that the function $\phi = \phi(t, x, y, z)$ is a function of all the coordinates.

(3) The spatial metric on the surface of a sphere with fixed radius $r = a$ has the form

$$ds^2 = a^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \text{ or, equivalently } (g_{ij}) \to \begin{pmatrix} a^2 & 0 \\ 0 & a^2 \sin^2\theta \end{pmatrix}$$

Find all the Christoffel symbols for this sphere using the coordinates (θ, φ).

(4) 5.15 Practice with taking covariant derivatives. Schutz uses ";" for the co-variant derivative so that $\nabla_\alpha V^\beta = V^\beta_{;\alpha}$. Don't recompute the Christoffels for polar coordinates. Instead use your notes or Eq. (5.45).

(5) If you have access to a laptop with enough free memory install Mathematica. If you do not have access, then log on to the pguest account in the area outside my office. In either case run this command: `Plot [ 1 - (Sqrt[r^2 + 4])^(-1), {r, 0, 8}]` and submit the result.

(6) (Optional) 6.39 On Lie derivatives - useful for those thinking of doing more GR after this semester.