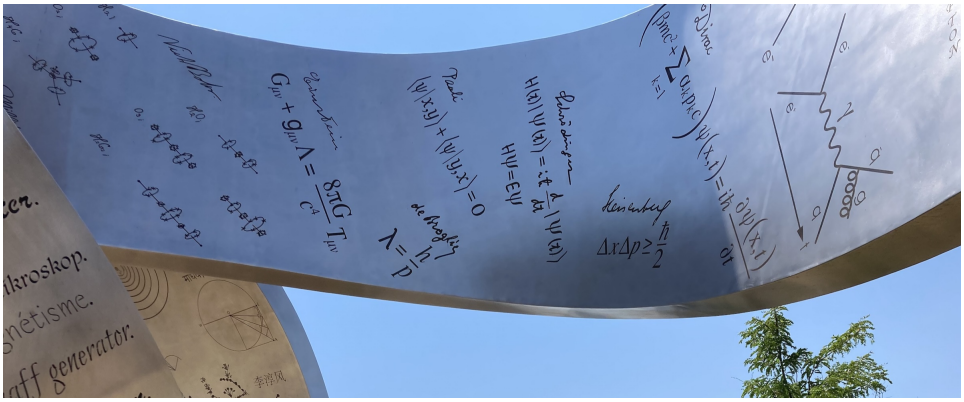
General Relativity (PHYS 325) PS 9

Spring 2026 v1.5



- There's a fun sculpture outside the visitor center at CERN. There's GR!

Now that we have found Einstein's equations this week we start to study solutions. First black holes! We'll continue to explore the Schwarzschild spacetime - first orbits and an upcoming observation and then the geometry.

I like slightly different notation than that what is used in Schutz for oribits. To translate between your notes and Schutz note that

$$\tilde{E} = e \text{ and } \tilde{J} = \ell$$

Reading:

- Schutz Chapter 10 sections 1 - 2
- Schutz Chapter 11 on BH's to page 328 - we will not launch into the Kerr solution (rotating BH's yet)* *starting on page 299 Schutz uses slightly different notation than we'll use in class so you may wish to skim Schutz's derivation of the orbit equation.*

Problems:

(1) (2 pts.) Let's return to the spacetime metric in PS 6, but now including the angular coordinates. Here is the metric,

$$ds^2 = -\left(1 - \frac{1}{\sqrt{r^2+a^2}}\right) dt^2 + \left(1 - \frac{1}{\sqrt{r^2+a^2}}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

where a is a constant and $d\Omega^2 = d\theta^2 + \sin^2(\theta) d\varphi$.

(a) Find the Christoffel symbols. Using Mathematica and the einstein.nb notebook is fine. You can also write them down from previous work, with a little explanation.
(b) Find the Riemann tensor.
(c) Find the Einstein tensor.
(d) Is this a vacuum spacetime? Explain why or why not.

(2) You find yourself joining Spaceman Spiff on a trip to compact spherical object of mass M. How far does Spiff's spacecraft travel from a spherical shell of area $144\pi M^2$ to a spherical shell of area $400\pi M^2$?

(3) 10.3 on energy

(4) Read section 11.1. Summarize the whole section being sure to complete the unit conversion to find the Schwarzschild radius of a solar mass black hole to three significant figures and the (surprising!) density result. Please show all your work.

(5) (1/2 pt.) 11.1 Just write down the result from problem (4), where all the work is done.

(6) On the effective potential discussed in class.
   (a) Plot the effective potential $\mathcal{U}_{eff}$ for massive particles around a Schwarzschild black hole for orbits with $\ell = 4.4M$. Label your axes and ensure that your plot shows the complete behavior of the effective potential.
   (b) Describe the possible orbits.

(7) DELAYED TO NEXT WEEK 11.24 (2 pts.) Ignore the extra term of dr^2 in equation (11.127). It is a typo.