The Importance of Mathematical Relationship Formation in Math Word Problem Solving:

An Eye Movement Analysis

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Abstract

An investigation into the importance of mathematical relationship formation in word problem solving sought to identify ways in which mathematical reasoning and achievement can be improved. The investigation broke problem solving into a series of steps with the hopes of facilitating solution accuracy though a proposed cognitive model which emphasized the importance of relative set size identification and relationship formation in the problem solving process. Participants were presented with a series of one, two and three part word problems aimed at examining the importance of each step, while their eye movements were monitored in order to identify patterns in attentional allocation during the process. It was hypothesized that following the steps in the proposed cognitive model would result in higher problem solving accuracy and that high accuracy problem solvers would devote more attention to the relational statement of the problem than low accuracy problem solvers. Results supported the proposed cognitive model, suggesting that problem solving accuracy is facilitated through model application. Finally, eye movement analyses revealed that high and low accuracy problem solvers focus their attention in different places within the problem text; specifically high accuracy problem solvers attend more to the question than low accuracy problem solvers. Trends identified in this study can be applied to education in order to improve the math problem solving skill of students.

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Knowledge of mathematics and an ability to integrate mathematical skills into everyday life is becoming increasingly important in our industrialized society (Roman, 2004). Math education is critical in preparing young people for the mathematical and problem solving challenges that they will face as adults. Compared to many industrialized nations around the world, students in the US are lagging behind in mathematics achievement (Mayer, Tajika & Stanley, 1991). This indicates a real need for examining the math education system in the US.

Mathematics education can be thought of as the teaching of two important topics: the execution of procedural skills and the solving of word problems (Nesher, 1986). A word problem sets up a situation in which one or more unknown variable(s) are defined in terms of a known variable(s). Often, word problems take the form of a short story, such as the example in Figure 4. Math educators are responsible for teaching students how to perform arithmetic operations as well as teaching them how to apply their arithmetic skill to real world quantitative problems. Interestingly, students who excel in arithmetic skill might not be successful problem solvers (De Corte, Verschaffel & Pauwels, 1990), indicating a need for students to develop a way of thinking about mathematics that is not simply a rote memorization of arithmetic facts.

One aspect of problem solving is the progression that one must go through in order to arrive at a solution. For example, when presented with a problem, one must first decide what is being asked, then the appropriate information must be selected from the problem in order to create a solvable mathematical relationship, and finally a series of computations must be performed in order to arrive at the numerical answer. Often, children are told what types of problem solving skills to use in given situations, making it difficult for them to make problem solving decisions when they are unprompted (Swan, 2004). One focus of math education should be to teach the skills necessary to facilitate independence and success in problem solving.

This study will investigate the components of problem solving as they relate to mathematics performance in order to determine sources of difficulty in the problem solving process. In particular, the process of mathematical relationship formation will be examined in order to understand its contribution to problem solving accuracy. Additionally, an examination of the processes that facilitate mathematical relationship formation will be conducted. Through eye movement analyses, the attention and focus of participants will be monitored as they attempt to solve various math word problems. Eye movement analyses operate on the premise that eye fixations correspond to attention allocation. In this study, it is assumed that people will spend a greater proportion of time looking at the components of the word problems to which they are attending. By identifying common areas of attention and inattention associated with problem solving difficulty, we will be better equipped to understand the process of problem solving and will eventually be able to use this knowledge to improve math education.

This paper will focus on the processes involved in mathematical relationship formation. Mathematical relationship formation is the process through which the text of a problem is transformed into a numerical mathematical relationship or equation. This relationship generally establishes a link between different variables in the text, often defining the value of one variable in terms of the value of another. Because equations are the primary way to represent mathematical text as numbers (Hinsley, Hayes & Simon, 1977), the process of mathematical relationship formation can be used to monitor comprehension and to trace the solution process involved in math word problem solving.

Cognitive Components of Mathematical Problem Solving

According to Nesher (1986) two key topics exist in math education. Students must learn to execute procedural skills and they must learn to solve word problems. The difference here lies in the task itself: procedural skills are generally used to solve computational problems while a different type of thinking must be employed when students are trying to solve a word problem. (For the purposes of this paper, we will refer to procedural knowledge as computational skill.) The solving of word problems draws on both problem solving skill – how well a problem solver can interpret and set up the text of a word problem for solution computations – and procedural skill, as the process generally requires some sort of computation in order to arrive at a solution. In this way, problem solving accuracy is strongly dependant on correct procedural knowledge, but requires an additional ability to understand and interpret the logical structure of a given text. Because both computational skill and problem solving skill can influence the accuracy of solutions, it is important to differentiate between the processes involved in each in order to better understand the problem solving process as a whole. For the purposes of this paper, problem solving success will be defined as the arrival at an accurate solution.

Computation vs. Problem Solving. The computational skills introduced by Nesher are necessary to perform arithmetic computations. These types of skills can be further divided into procedural and propositional (or declarative) representations of knowledge (Nesher, 1986). A propositional representation of knowledge is a factual statement that is readily retrieved from memory. For example, it is easy for most adults to know, without performing any computation, that 3+2 = 5 because this is a math fact that has been practiced and memorized, and can be easily retrieved from memory. However, when presented with (3+2) (7-4) = ?, adults must draw on some sort of procedural knowledge that tells them to first perform the operations within the parentheses and then to multiply the two results. In this case, the values inside the parentheses are relatively simple and will most likely be computed using a propositional representation of knowledge, but the process which must be followed in order to perform the entire computation accurately requires a procedural representation of knowledge. Regardless of the processes required in computation, it is important to consider that arithmetic ability is only one component of solving word problems, and therefore only contribute so much to solving word problems successfully.

Consequently, is possible for a person to be very good at computation but perform quite poorly on tests of problem solving ability (DeCorte, et al., 1990) resulting in inaccurate solutions. This is consistent with the notion that computational skill and problem solving ability constitute distinctly different domains, and skill with one does not necessarily imply skill with the other. A study conducted by Mayer, et al. (1991) specifically discriminates between problem solving ability and computational skill in order to uncover differences between the math abilities of American students and those of Japanese students.

In order to address the notion that problem solving ability is separate from, but dependent upon, computational skill, Mayer, et al. (1991) administered two different math tests to the student participants, one testing computational skill and the other testing problem solving ability. As many studies comparing the mathematics performance of Americans to performance of children in other industrialized nations have found, this study revealed that Japanese significantly outperform their cohorts on tests of both computational ability and problem solving skill (Mayer, et al. 1991). This suggests an independence of the processes involved in computation and the processes involved in problem solving. Based on these findings, researchers should first take into account students' computational ability in order to isolate and more directly compare problem solving ability.

Mayer, et al. accounted for computational ability by placing students into different achievement levels based on their performance on the math computation test. They then compared students' problem solving abilities within the similar computational ability groups. This allowed for comparisons to be made between students of similar computational ability, thus decreasing the variation in performance due to computational ability, and allowing comparison of problem solving ability alone. Within the achievement levels, they found that American students performed better than their Japanese counterparts on problem solving tasks (Mayer, et al. 1991). This finding provides additional support for the independence between computational ability and problem solving, further supporting suggestions that both processes should be examined separately in the solution process.

One way to investigate the delineation between the components is to eliminate certain requirements of the problem solving task. This can be done by isolating either the problem solving component or the computational component. Hagarty, Mayer and Green (1992) eliminated the computational component by asking students to merely state a plan for how they would solve the problem rather than carry out the computation required to arrive at a solution, thus allowing researchers to learn more about the other phases of problem solving.

Another way to account for computational ability is to administer a computational skills test in order to be able to associate computational skill with varying degrees of problem solving ability (De Corte, et al., 1990; Mayer, et al., 1991).

According to Nesher, the best way to test for understanding in mathematics is through the use of word problems since the solving of math word problems requires proficiency in multiple domains (Nesher,1986). Neither arithmetic skill nor problem solving ability are sufficient for solution accuracy when independent of the other, therefore a firm grasp of problem solving demonstrates competent computational skill as well as developed problem solving techniques (DeCorte, et al., 1990).

Types of Word Problems

Math word problems can vary greatly in terms of their structure, their complexity and the type of situation they are addressing. Structurally, a standard problem typically consists of three parts: an assignment sentence, a relational sentence and a question (Mayer, 1981, 1982 as cited by Lewis and Mayer, 1987). An assignment sentence gives a numerical value to a variable: *Elisabeth has 5 apples.* A relational sentence defines one variable in terms of another: *Stephanie has 3 more apples than Elisabeth.* Finally, the question asks the problem solver to determine the value of an unknown variable: *How many apples does Stephanie have?* In order to solve the word problem, students must be able to interpret each of these statements and integrate their meaning into a solvable format. The procedure required to do this is often quite difficult, requiring students to convert the text of the problem into a numerical system: a mathematical relationship or equation.

Nesher (1986) proposes three different types of mathematical situations; that is, three ways in which the word problem can be set up. Each situation – Dynamic, Static and Compare – require a different approach to its solution. As defined by Nesher, a dynamic change situation describes changes to an initial situation. For example, *Sara had 7 pens. She gave two away. How many pens does Sara have now?* A static situation addresses a relationship between sets with no change occurring in the text. For example, *There are 4 spoons and 5 forks on the table. How many eating utensils are on the table?* Finally, a comparative situation describes a relationship between sets: *Laura has 4 books. Alexis has 3 more than Laura. How many books does Alexis have?*

Interestingly the different types of problem situations elicit differing success rates for problem solvers (Nesher, 1986) possibly indicating relative differences in complexity. In both Israeli and United States samples it was found that Compare problems elicit successful responses only 60% of the time, while Dynamic change situation and Static situations elicit successful responses 86% and 80% of the time respectively (Nesher, 1986). Hegarty, et al. (1992) suggests that the relatively poor performance on Compare problems might reflect an inability of the problem solver to create accurate mathematical representations of the text. That is, students may have more trouble transforming the text into a mathematical relationship when the text takes certain forms, such as that of Compare problems.

Phases of Problem Solving

Problem solving, unlike arithmetic, is not an automatic phenomenon, but rather a progression that requires the integration of more complex processes. Tajika (1994) proposed that the process of problem solving involves four cognitive phases: translation, integration, planning and execution. The translation phase generally refers to the initial reading of the problem during which the problem solver creates an internal representation of the individual propositions in the text, relying heavily on reading comprehension. The integration phase requires the problem solver to select the relevant information from the text in order to coherently represent the problem as a whole. This often involves the identification of relationships between variables in the text. During the planning phase, the problem solver generates a plan for solving the problem by breaking it down into a series of steps, often requiring the creation of mathematical representations of the text. Finally the arithmetic computations are carried out during the execution phase (Tajika, 1994). Clearly the execution phase makes use of a students' computational skill, while the other three phases draw on problem solving ability.

Tajika (1994) suggests that the four phases of problem solving occur in serial order: first translation, then integration, then planning and finally execution; however it is possible that these components merely interact with each other at various stages in problem solving. If phases do in fact proceed serially it would be relatively easy to test the impact of each component on problem solving ability, but if these phases overlap or occur simultaneously it is much more difficult to discriminate between the various phases.

Since difficulty in problem solving is due to under developed skills in one or more phases of the solution process (Osterroth, 1994), it is important to be able to discriminate between the various phases of problem solving so that researchers can focus their attention on difficulties that arise during individual phases or processes. Attention to a particular phase or process can give researchers clues about the progression used to arrive at a solution, and to the loci of common errors. This can contribute to the development of methods designed to decrease or eliminate errors in that particular phase and ultimately in the problem overall.

For example, there is evidence to suggest that high achieving math students out perform low achieving math students during the integration phase: that is, when both relevant and irrelevant information are present high achieving math students are generally better at selecting the relevant information from a problem text (Cook & Rieser, 2005). Investigation into this discrepancy will yield clues about the superior processing techniques used by high achievers with possible educational implications.

Translation. The first phase of problem solving introduced by Tajika (1994) is Translation: the initial reading of the problem during which the problem solver creates an internal representation of the individual propositions in the text. This phase is often defined by researchers as the initial reading of the problem and can be measured as the time, as monitored by eye movement analysis, taken to read through the problem once (DeCorte et al., 1990; Hegarty, et al., 1992; Terry, 1992). By defining the phase in terms of a pattern of eye movements, researchers are able to isolate the translation phase for further inspection. This allows for an investigation of the cognitive processes occurring during individual phases.

Terry (1992) examined the first reading (translation) phase of math word problems in college students in order to discern patterns of attention to numbers in mathematical text. By monitoring eye movements, he found that almost everyone attended to the numbers during the initial first reading, but for the most part, the exact identity of numbers in the text could not be recalled (Terry, 1992). Terry hypothesized an explanation for this phenomenon by suggesting that the first reading serves as a way for subjects to think about the problem without paying close attention to specific numerical details. Perhaps during the translation phase, problem solvers merely gain a general understanding of the problem and the relationships that exist within the text.

Execution. The way in which the translation phase has been defined in previous research makes it relatively easy to separate it conceptually from the other three phases of problem solving. The other phase, which is relatively easy to isolate, is the execution phase. Defined by Tajika (1994) as the period during which arithmetic computation is performed, researchers can identify this phase by observing when computation begins. As discussed earlier, a student's computational ability can act independently of his or her problem solving ability with expertise in one area not necessarily indicating expertise in the other. In this way, many researchers think of the execution phase as being unrelated to the understanding of the problem itself (Mayer, et al., 1991).

Integration and Planning. The two intermediate phases of problem solving: the integration phase and the planning phase, are generally harder to isolate from the entire problem solving process. Because the integration process requires problem solvers to select the relevant information from the text in order to coherently represent the problem as a whole, and the planning component requires the problem solver to generate a plan for solving the problem by breaking it down in to a series of steps (Tajika, 1994), it seems logical that these two phases occur relatively simultaneously.

In order to investigate the difference between the integration phase and the planning phase of problem solving, Tajika (1994) used simple word problems and multiple choice questions aimed at guiding students to focus their cognitive processing on the phase being targeted. For example, a question focusing on integration would ask students to select the numbers (or words) necessary to solve the problem, and a question focusing on the planning component would ask students what operations they were planning to use in order to solve the problem. The results of this research suggested that high and low math achievers differed most significantly during the integration phase of problem solving (Tajika, 1994). These findings demonstrate the importance of information selection and problem representation in the problem solving process. It is important to remember, however, that weakness in a single phase of problem solving can impede accurate solution formation.

Eye Movement Analysis and its Contribution to the Study of Math Word Problems

Eye movement analysis is a process by which the eye movements of a subject can be examined in relation to a visual stimulus. Generally this technology involves some sort of camera that senses the eye's movements in relation to a visual field. A computer then integrates the eye movements with the visual field giving researchers information about where a person is fixating at any given time.

Eye movements are not smooth sweeps across a visual stimulus; rather they are a series of saccades and fixations. A saccade is a rapid movement of the eye from one point to another and a fixation is a pause in this movement on some part of the stimulus. During fixations the eye takes in visual information from the stimulus, while the saccade shifts the fixation locations (Ashcraft, 2002). Evidence suggests that attention can have control over eye movements (Theeuwes et al., 1998 as cited by Ashcraft, 2002).

The idea underlying eye movement research is that eye movements are closely associated with attention. This relationship has been examined in multiple contexts; Posner, Cohen and Rafal (1982) found that people respond faster to a visual stimulus when their attention is cued to its location. That is, visual response time -- how long it takes to shift the eyes to a stimulus -- is slower to uncued stimuli than to cued stimuli. This demonstrates that there is a link between attention and visual orienting.

The connection between attention and visual orienting has important implications for understanding the cognitive processes involved in reading: if people are attending to some word or phrase, they are most likely looking at it as well. Interestingly, Posner and Cohen (1984) discuss research that has demonstrated that the effective visual field -- the area to which people are actively attending -- in reading differs depending on the direction in which the language is read. In languages read from left to right the effective visual field is larger to the right of fixation than to the left, and the opposite is true in languages read from right to left (Posner & Cohen, 1984). This demonstrates how, in reading, shifts in attention occur in connection with the movement of the eyes supporting the idea that eye fixations are closely associated with attention.

Analyzing eye movements in the reading of a math word problem can help researchers differentiate between the different phases of problem solving. As previously defined, the translation phase generally refers to the initial reading of the problem during which the problem solver creates an internal representation of the individual propositions in the text (Tajika, 1994). Researchers can then use eye movement analysis to determine how long problem solvers take to complete the translation phase. DeCorte, et al. (1990) defined the start of the translation phase as the time when the problem is first presented and the end of the phase as the time when the problem solver completes the initial first reading, taking into account the sequence of consecutive fixations. Generally, researchers look to see that sequential eye movements occur in the reading direction, indicating the reading of, rather than the searching for, information (DeCorte, et al., 1990).

Eye movement analysis can be used to determine how people allocate their attention to certain visual stimuli, including math problems. Fry (1987) used eye movement analysis to detect the ability of problem solvers to choose the relevant information from a problem text. Eye moment analysis suggested that students who were more successful in extracting the relevant information also spent more time fixating on the relevant information (Fry, 1987). This finding further supports the theory that attention is strongly liked to eye fixations; those who selected (and thus attended to) the appropriate information from word problems spent a longer time fixating on it.

Math word problems constitute an area in which problem solvers must pay attention to words (context), numbers (values), and relationships between numbers (operators). By examining eye movements, researchers can gain a better understanding of the cognitive processes involved in solution formation as they relate to the allocation of attention to various types of information presented in word problems.

Factors that affect problem solving

Math and Reading Ability. The solving of math word problems obviously requires the problem solver to first read and comprehend the problem before attempting to answer it; therefore it is easy to see that reading ability plays a critical role in math word problem solving. Fuchs, Fuchs and Prentice (2004) claim that the research on mathematics performance often fails to control for reading ability. For example reading ability can have a detrimental effect on problem solving speed for low ability readers. Cook and Rieser (2005) discovered that low achieving math students read significantly fewer words per minute in word problem solving tasks, thus slowing their time to solution. Reading comprehension skills most likely have a similar detrimental effect on word problem solving ability. Also, as discussed previously, computational skill can also influence problem solving performance (Mayer, et al. 1991; De Corte, et al. 1990). Therefore it is important to consider both reading ability and computational ability when assessing student performance on math word problems.

Information selection. As is evident from the phases of problem solving described by Tajika (1994), the selection of the important information from a math word problem, occurring during the integration phase, is critical to arriving at an accurate solution. Word problems consist of both words and numbers, and problem solvers need to be able to differentiate between the relevant and irrelevant information in the problem text. For the purposes of this study relevant information is that which is essential to the solving of the problem, whereas irrelevant information may provide context but is not necessary for solution.

Cook and Rieser (2005) compared the visual scan patterns of high and low achieving elementary school math students as they worked to complete math word problems containing both relevant and irrelevant information with varying degrees of difficulty. As would be expected, problems containing irrelevant information proved to be more difficult to solve than problems containing no irrelevant information. They also found that the presence of irrelevant information in the problem was more detrimental to performance in more challenging problems. This suggests that the presence of irrelevant information increases the demand placed on the working memory, and when coupled with a difficult or complex problem, this increased demand can be more detrimental to performance. Finally, high achieving math students were better able to select the relevant information in these more complex problems, perhaps indicating a superior ability to reduce the strain placed on working memory. This could signify that low ability math students are lacking efficient information selection strategies, supporting Tajika's (1994) finding that low achieving math students experienced significantly more difficulty during the integration phase of problem solving.

Cook and Rieser were also able to use their eye movement analysis to differentiate between different visual scanning strategies used to search for and ultimately select information. The three strategies identified were as follows: *number grabbing,* in which numbers are selected from the text with little or no consideration paid to their relationship to the problem's meaning; *simple comparison,* in which an attempt is made to discriminate the relevant and irrelevant information based on comparisons of text and numbers; and *question-guided comparison*, in which special attention is paid to the question in order to identify what types of information are being sought. It was discovered that question-guided comparison proved to be the most accurate strategy for the selection of relevant information, and this was the most popular selection strategy among high achieving students (Cook & Rieser, 2005).

Nesher (1986) provides a similar finding to that of Cook and Rieser, presenting the basic word problem as consisting of three strings or phrases (assignment, relation and question) each revealing a different type of information to the problem solver. As discussed earlier, the last phase is generally the question, which indicates the type of solution that is required. Nesher emphasizes the importance of reading and understanding the entire text before attempting to answer the problem since the question phrase has the potential to elicit very different interpretations of the problem situation. For example, the first two strings or phrases of a word problem may be: *1.) Twenty students entered the room for class at 9am. 2.) At 9:30 five students were summoned to the principal's office.* Following these strings are two types of possible questions: a subtraction question: *How many students were left in the classroom?* or an addition question: *How many times did the door to the classroom open?* Clearly it is important for the reader to attend to and understand what is being asked in the question in order for them to create an accurate representation of the problem.

Providing further support for the importance of relevant information selection, Moreau and Coquin-Viennot (2003) investigated the information selection patterns of high and low achieving fifth grade students. High math ability students were better at selecting the information relevant to solving the problem than low math ability students

(Moreau & Coquin-Viennot, 2003). Clearly the selection of appropriate information is important to the solution process: selection of irrelevant information will be detrimental to the accuracy of the solution while selection of relevant information will provide the base necessary to continue with the problem.

Structure of the Word Problem. The structure of a word problem can have a substantial influence on its solvability. As previously mentioned, word problems are generally composed of a series of sentences assigning values to different variables and representing relationships between these variables, in addition to a question requesting the unknown value of a specific variable. Different types of word problems can vary in terms of their structures; a static situation is presented differently to a dynamic change situation or a compare situation. Interestingly, within each problem category, the structure of the word problem can still be manipulated. For example, in dynamic change problems, the unknown quantity can be presented as the start set, the change set or the result set (De Corte, et al., 1990). For example, the dynamic change problem, *Sara had 7 pens. She gave two away. How many pens does Sara have now?* represents the presentation of the unknown quantity as the result set; problem solvers need to determine how many were resulting after the change occurred. In contrast, the problem, *Sara gave two pens away. She was left with 5 pens. How many pens did she start with?* represents a problem in which the unknown quantity is presented as the start set.

Because problem solving requires readers to create a mathematical representation of the problem, decisions must be made as to what information is important and what operations will be used. According to De Corte, et al. (1990) the different types of word problems vary in terms of the complexity of processing required in order to arrive at an accurate problem representation. Because compare situations appear to be the most difficult for problem solvers (Lewis & Mayer, 1987), focus will be placed on examining their structural differences.

Lewis and Mayer (1987) examined a consistency effect in the solving of compare word problems. Consistency refers to the way in which the information is presented in the text of the word problem: the relational term (or operation) implied in the relational statement can either be consistent or inconsistent with the operation required to solve the problem. For example, a relational term such as “less than” implies subtraction, and a relational term such as “more than” implies addition. Lewis and Mayer examined the effects on solution accuracy of implying an operation consistent with the operation required in the question (a consistent problem) or implying an operation inconsistent with the required operation (an inconsistent problem). Examination of the construction types reveals that in the inconsistent problems the unknown quantity is represented as the object of the relational sentence, whereas in consistent problems, the unknown quantity is represented as the subject of the sentence.

Results showed that students had more difficulty with inconsistent than consistent problems, with the majority of errors being reversal errors (Lewis & Mayer, 1987). A reversal error, as defined by Lewis and Mayer, was one in which the arithmetic operation necessary to solve the problem was reversed. It is hypothesized that this reversal error is due to a priming effect of the relational term in the relational statement on the required operation. For example, when presented with the problem: *Elisabeth has 5 apples. Elisabeth has 3 <u>more</u> apples than Stephanie. How many apples does Stephanie have?* problem solvers are primed to think about addition because the relational term is *more* in the relational statement, however solving the problem requires subtraction. This creates confusion and subsequent difficulty in creating a mathematical representation of the relationship between the variables in the problem.

The major difference between the inconsistent and consistent problems was in the construction of their relational statements, which differed in two ways: consistency of the statement with the required operation, and the representation of the unknown value as either the subject or the object of the relational sentence. According to Lewis and Mayer, the ideal structure for the relational statement in a compare word problem involves consistent language, in which the operation implied in the relational statement matches the operation required for solution, and the unknown variable is presented as the subject rather than the object of the relational sentence. They hypothesize that when relational statements are not in this "ideal" structure, problem solvers must rearrange the statement in order to make it fit the desired format (Lewis and Mayer, 1987).

This concept is supported by Hinsley, Hayes and Simon (1977) who suggest that people try to fit mathematics problems into a "normal schema;" one that is familiar or easier to deal with. Lewis and Mayer's "ideal" structure is cognitively easier to comprehend because it is operationally consistent (the implied operation is the same as the required operation). Because difficult tasks put greater strain on the working memory (Kintsch & Greeno, 1985), the rearrangement of word problems to fit an "ideal" form requires additional processing and time. This can result in the potential for more errors, such as the reversal errors discovered by Lewis and Mayer (1987). Such errors can be classified as a general difficulty in creating mathematical representations of word problems.

Verschaffel, De Corte and Pauwels (1992) examined Lewis and Mayer's consistency hypothesis using eye movement analysis in order to provide further information about the processes involved in their model. They found that students spend more time fixating on the relational sentence in inconsistent problems than in consistent problems indicating that this relational statement was a source of confusion. This suggests that subjects may have been manipulating the sentence into a form that was more compatible with the structure of the "ideal" relational statement. Interestingly, this trend was true only when the problem created sufficient cognitive demand on the problem solver (Verschaffel, et al., 1992). This indicates that the structure of the relational statement has the potential to place additional cognitive demands on problem solvers, which becomes visible when the strain on working memory is already high due to problem difficulty.

Creation of mathematical relationships from the text. The job of the problem solver is to create a mathematical relationship between variables in the problem in order to prepare for the execution phase of problem solving. Certain words and phrases can be helpful in aiding the reader to create accurate mathematical relationships; however this skill is often quite difficult to master.

In an earlier example, the word problem: *Elisabeth has 5 apples. Stephanie has 3 more apples than Elisabeth. How many apples does Stephanie have?* relates the number of apples held by the two people named in the problem. This can be thought of the number of objects (apples) in two different sets (where the set is denoted by a person's name). In order to answer the question, problem solvers must first generate a mathematical relationship between the two variables, in this case Elisabeth and

Stephanie. Generally, the relational sentence provides the key for the creation of the mathematical relationship. In the above relational sentence, the important things to note are the type of relationship: "more than," and the direction of the relationship, Stephanie has the larger value. In this case the mathematical relationship would be: E+3=S, where E represents Elisabeth and S represents Stephanie.

The process of forming mathematical relationships can prove to be quite difficult, even in adults. Soloway, Lochhead and Clement (1982, as cited by Lewis & Mayer, 1987) asked college students to write mathematical equations that represented relational statements. Relational statements such as: *Dorothy has 3 times as many books as Courtney,* should be represented with 3C = D, where C represents Courtney and D represents Dorothy. However, it was found that approximately one third of college students created an inaccurate mathematical representation of the relationship such as 3D=C. This indicates a need to examine the process through which mathematical relationship formation occurs during problem solving.

As previously noted, the creation of accurate mathematical representations of problem text is crucial to problem solving success (Hinsley, et al., 1977). Kintsch and Greeno (1985) claim that problem solvers use these mathematical representations to choose the appropriate operations for calculations. In creating these representations, Kintsch and Greeno suggest problem solvers must pay attention to set representations and set relations; in other words problem solvers must develop a set schema. A set schema is a way in which problem solvers can assign values to different objects in a problem in order to understand the relationship between objects or variables. Problem solvers must be able to identify and differentiate between sets, associate quantities with sets and recognize relationships between sets (Kintsh & Greeno, 1985).

Generally, the relationship between sets can be established prior to reading the question. (So if the question is inconsistent with the relational statement, relationship formation may be more difficult as mentioned earlier by Lewis and Mayer's (1987) consistency effect.) For example, a typical assignment sentence and relational sentence can be represented as: *Sara has 2 rings. Mary has 3 more rings than Sara.* Here, problem solvers must recognize the objects in the set: in this case rings; quantify the sets: the known set has two members; and specify the sets: one is named Sara and the other is named Mary. Finally, the problem solver must relate the sets to one another; that is the problem solver must define the unknown quantity of the set Mary in terms of the known set Sara: *Mary = Sara+3* Accurately defining this relationship is critical to successful problem solving.

The Present Study. The transformation of the text of a word problem into a numerical mathematical relationship is a critical component of successful problem solving. Information required for this transformation is generally contained in the relational statement. This study will examine the process through which problem solvers transform word problem text into a mathematical representation of the variables in the problem, with focus placed on the transformation of the relational statement.

Through the analysis of eye movements, the association between fixation patterns within the problem and accurate mathematical relationship formation will be investigated. By examining where subjects fixate during a relationship formation task, we will be able to infer the focus of their attention. Time spent fixating on specific locations as well as the number of times a location is fixated will be monitored.

Because the creation of a mathematical relationship requires an understanding of the relationship between variables in the text, we will investigate components of this understanding in a variety of ways. First, the process of mathematical relationship formation will be investigated as it exists in a typical word problem; the creation of an equation which defines an unknown variable in terms of a known variable. Next, examination of how people determine the relative size of sets will be conducted; that is, whether it is easier for people to simply recognize a larger (or smaller) set than it is for them to determine a mathematical relationship between sets. According to Michie (1985) ordinal position identification, or the ability to determine the relative size of sets, is developed early in life, so we will test to see if this is a more automatic (and accurate) process compared to relationship formation.

The investigation will also break problem solving into a series of steps with the hope of facilitating successful problem solving through a proposed cognitive model. This model suggests that accurate problem solving of compare word problems occurs through a three step process of 1.) set size identification, 2.) relationship formation and 3.) solution achievement (see Figure 1). By testing for facilitation effects of earlier steps on later steps, investigation of the efficacy of this model will be conducted. To do this, subjects will first be prompted to identify the relative size of a set (larger or smaller than the other(s)), to then create a mathematical relationship between those sets, and finally to solve the problem.

Hypotheses. Accurate mathematical relationship formation is critical to successful problem solving (Hinsley, et al., 1977; Kintsh & Greeno, 1985), therefore it is predicted that there will be a strong correlation between mathematical relationship formation and problem solving success. Furthermore, it is hypothesized that mathematical relationship formation prompts will facilitate subsequent problem solving accuracy. That is, it is predicted that people will perform better on word problems in which they are prompted to form a mathematical relationship between the variables in the text prior to computing a solution.

Next, since relational terms, such as “more than” or “twice as many,” are important in mathematical relationship formation (Lewis & Mayer, 1987; Verschaffel et al., 1992), it is predicted that problem solvers who are better at creating accurate mathematical representations of the text will spend more time fixating on the relational statement than those who have difficulty with relationship formation.

Additionally, since ordinal position identification is developed early in life (Michie, 1985), it is hypothesized that mathematical relationship creation will be more difficult for the average problem solver than relative set size recognition. Therefore, it is predicted that subjects will be better at determining the relative size of sets than they will be at creating accurate mathematical representations of the text.

Finally, since inaccurate mathematical relationship formation often creates misrepresentations of the relative size of sets (Soloway, Lochhead & Clement, 1982, as cited by Lewis & Mayer, 1987), it is hypothesized that problem solvers will be able to form more accurate relationships between sets if they are first prompted to focus on the relative size of the sets. That is, it is expected that set size identification prompts will facilitate relationship formation accuracy; with people forming more accurate mathematical representations when first prompted to identify the relative size of the sets in question.

Method

Participants

Subjects, selected from the Hamilton College graduating class of 2007, consisted of six males and 19 females, with a mean age of 20 years. The selection process was based on the students' score on the quantitative literacy exam (Q-lit exam), administered during their freshmen year orientation. Students with Q-lit scores in the upper and lower quartiles of their class were randomly selected within their group to represent students with high and low math ability respectively. The recruitment process consisted of emails and phone calls requesting participation. Participation was voluntary, and a $5 gift certificate to a coffee shop on campus was offered as compensation. Confidentiality was ensured by pairing each subject name with a subject number which was used as the only identifier throughout the experiment and analysis.

Apparatus

Eye Tracking. A head mounted ASL (Applied Science Laboratories) 501 eye tracking system was used to monitor eye movements while subjects solved a series of math word problems. The ASL 501 eye tracker operates with a Magnetic Head Tracking unit (MHT) and an Eye Head Integration System (EHI) which work together to compensate for head movements. That is, the MHT and EHI system account for head movements during data collection so that subjects are able to move their head freely during the experimental procedure. Eye movement data was collected with a high speed camera which took a reading of the eye position approximately 120 times per second. The collected data can be used to determine the location and duration of eye fixations as well as the sequence of these fixations. Eye movement data is only meaningful when it occurs within a specified visual field. In this study, eye movements were monitored in relation to a computer screen on which the stimulus was presented.

Subjects were seated so that they were no more than three feet away from the MHT and so that they could comfortably view the computer screen, press keys on the keyboard and work with the answer sheet; they were generally less than two feet away from the computer screen. The eye tracking head piece was positioned so that the camera was over the subject's left eye (see Figure 2).

Stimuli

The visual stimulus was presented to subjects on a computer monitor using PsyScope, a program commonly used in cognitive research at Hamilton College. The stimulus consisted of a series of 18 math word problems, each presented in one of three formats: simple question problem (Q), relationship-question problem (RQ) and set size-relationship-question problem (SRQ); there were six Q, six RQ, and six SRQ problems in each presentation. Each of the 18 problems started with an assignment sentence (AS) and a relational statement (RS), which together made up the initial conditions (IC) of the problem. Each initial condition was then followed by either one, two or three questions in the Q, RQ and SRQ problems respectively (see Figure 3). Some responses were multiple choice, while others were free response.

Simple Question Problem. The format of the simple question problem was an initial condition followed by a solution question (Q) (see Figure 4). This was designed to represent the form of a traditional compare problem, with only a solution question aimed at assessing problem solving accuracy. The solution question problem required a free response answer.

Relationship-Question Problem. The relationship-question problem was designed with an initial condition followed sequentially by two questions: a relationship question (RQ1) and a solution question (RQ2) (see Figure 5). The relationship question, which required a multiple choice response, was designed to investigate the participants' ability to convert a textual statement into a mathematical relationship. The solution question was, again, aimed at assessing problem solving accuracy and again required a free response solution. The relationship question (RQ1) was presented prior to the solution question (RQ2) in order to determine its effect on solution accuracy.

Set Size-Relationship-Question Problem. The format of the set size-relationship-question problem was an initial condition followed sequentially by three questions: a set size identification question (SRQ1), a relationship question(SRQ2), and a solution question (SRQ3) (see Figure 6). The set size question was designed to prompt participants to examine the relative size of the sets in the problem prior to creating a mathematical relationship between them. The relationship question and the solution question were presented for the same purposes as in the previous problem types. Both the set size question and the relationship question required multiple choice responses, while the solution question required a free response.

Presentation. Problems with more than one part were presented sequentially, and participants advanced from one part to the next by pressing the spacebar. For multi-step problems, each step was added sequentially to the computer screen when the participant clicked the spacebar. For example, for SRQ problems (like the one shown in Figure 6), participants first saw the initial conditions and part (a). When the spacebar was pressed, part (b) was added to the screen, and finally part (c) was added. This allowed participants to look back at preceding parts as they solved multi-step problems.

There were 18 different initial conditions, each of which was used in each of the three possible formats (Q, RQ, SRQ) and presented in three different, but balanced PsyScope presentations. Each presentation was counterbalanced across participants for format, difficulty and required operation, and the problems were presented to the participants in a random order. The problems ranged in difficulty and in required operation: there were six easy multiplication, six easy division, three difficult multiplication and three difficult division. Easy problems required only one step, and difficult problems required two steps.

A practice session of three problems – one simple question problem, one relationship-question problem and one set size-relationship-question problem was created to simulate the experimental task. The purpose of the practice session was to let participants adjust to wearing the eye tracking apparatus, to allow participants to become familiar with the format of the stimuli, and to give the experimenter an opportunity to ensure that everything was running smoothly and that participants were recording their answers in an appropriate fashion.

Procedure

Upon arrival to the lab, participants were introduced to the eye tracking apparatus and were given a brief overview of what the experiment would entail. Subjects were allowed to ask questions, and were then asked to sign a consent form (see Appendix 1).

The eye tracking apparatus was fitted to the participant's head and the experimenter calibrated the control computer to recognize the participant's eye.

Once calibration was complete, participants were presented with an instruction screen on the stimuli computer and were given a chance to ask questions. The experimenter then gave a few additional verbal instructions in order to clarify the procedure. Most importantly, participants were instructed to do as much mental work as possible, and to refrain from writing anything besides the answer on the answer sheet. This was done to discourage participants from spending too much time looking at the paper where their eye movement data could not be analyzed, and to encourage participants to look at the computer screen when they needed information. Participants were given space for scratch work if necessary.

Participants completed the practice session of three sample problems, recording their answers on the answer sheet provided. The experimenter then checked to see that the answers were recorded in the appropriate format, and participants were given another chance to ask procedural questions. Eye movement data collection was initiated when participants began the experimental test. Participants viewed the series of 18 math problems presented in random order, and recorded their responses on the answer sheet provided. When the test was completed, participants were verbally debriefed as to the purpose of the study and allowed to ask any further questions.

Response time data was collected for each question or question part, with response time being measured as the time between scene changes (space bar clicks), not in the time it took participants to record their answer. Answer sheets were scored based on accuracy alone, with correct responses valued at one point and incorrect responses valued at zero. Because some problem formats had multiple parts, each part was valued at one point, so that simple question problems were worth 1 point, RQ problems were worth 2 points and SRQ problems were worth 3 points. This allowed for a maximum of 36 points. The text of each word problem was divided into areas of interest (see Figure 7), and eye movement data was collected to provide information about fixation patterns within these areas.

Results

The purpose of this study was two fold; the first goal was to examine the importance of mathematical relationship formation in problem solving accuracy and the second was to explore the cognitive processing differences between high accuracy and low accuracy math problem solvers. Data was analyzed in two phases: the first phase investigated trends within the overall data with regard to problem solving accuracy, and the second phase investigated trends in a condensed data set with regard to eye movement patterns.

Initial Analyses of Accuracy

Initial examination of overall accuracy scores for all participants revealed a strong negative skew, with 50% of the participants missing 5 or fewer questions (see Figure 8). This finding suggests the presence of a ceiling effect on the experimental task, perhaps indicating that the task was too easy for the participant population. Additional examination of accuracy scores revealed that the mean score for every problem type fell within one standard deviation of a perfect score, with the exception of RQ1 (see Table 1 and Figure 9), further supporting the suggested ceiling effect.

Group Assignment

Participants were selected from the upper and lower quartile of scores on a quantitative literacy (Q-lit) examination that was administered at the beginning of the subjects' freshmen year in college, and were classified as "high math ability" or "low math ability" based on their performance. Initial analysis tested whether these labels of high and low ability could predict performance on the experimental task. Interestingly, those labeled as "high math ability" (as determined by the Q-lit exam) did not perform significantly better than those labeled as "low math ability" on measures of solution accuracy or reaction time (see Figure 10 for solution accuracy). Since participants were in their third year of college, it is possible that a measure three years prior to the experiment was no longer a good measure of their math ability.

In order to have groups of different ability on the experimental task, participant scores were re-examined and two new groups were defined; instead of "high math ability" and "low math ability," as defined by the Q-lit exam, subjects were classified as "high scorers" and "low scorers" based on their accuracy score (out of 36) on the experimental task. Participants with scores above the mean (M = 31.2, SD = 3.9) were labeled as "high scorers" and those with scores that fell below the mean were classified as "low scorers." This assignment left 12 participants in the "high scorers" group (M = 34.67 (SD = 1.37) and 13 participants in the "low scorers group" (M = 28, SD = 2.52).

As previously mentioned, the accuracy data overall, had a strong negative skew (see Figure 8), with the "low scorers" falling in a 9 point range (23 to 31), and the "high scorers" falling in a 5 point range (32 to 36) with five people achieving a perfect score. The skew of the accuracy data suggests the presence of a ceiling effect within the high scoring group; this decreased variability has the potential to make comparisons between the high and low scorers difficult.

Initial analysis comparing high and low scorers revealed significant differences in RQ1 accuracy ($t(23) = -5.5$, $p<.001$), RQ2 accuracy ($t(23) = -3.59$, $p<.01$), SRQ1 accuracy ($t(23) = -2.215$, $p < .05$), and SRQ2 accuracy ($t(23) = -3.8$, $p <.01$), with high scorers scoring significantly higher on each question type (see Figure 11). Marginal differences were seen for the other two question types (Q and SRQ3). Marginal differences were also seen between high and low scorers in response time, in the Q, RQ1 and SRQ2 problem conditions ($p < .10$) with high scorers performing faster than low scorers in every problem type. A complete listing of the descriptive statistics for accuracy and response time is provided in Tables 3 and 4 respectively.

Mathematical Relationship Formation Ability

The first hypothesis investigated a relationship between problem solving accuracy and mathematical relationship formation ability. There was, however, no significant correlation between mathematical relationship formation ability and problem solving accuracy as measured by Pearson Correlations between relationship questions (RQ1, SRQ2) and solution questions (Q, RQ2, SRQ3) (see Table 5). There was however a significant correlation between overall test score and relationship formation ability as measured by Pearson Correlations between RQ1 and test score ($r = .881$, $p < .001$) and between SRQ2 and test score ($r = .815$, $p < .001$), indicating that performance on the relationship formation questions significantly impact overall score. This impact may be due, in part, to the ceiling effect that was seen in the other question types. That is, overall accuracy scores essentially measured relationship formation ability, since there wasn't enough variability in the scores on other question types.

A t-test revealed that participants, overall, did in fact score significantly lower on the relationship formation questions ($M = 4.44$, $SD = 1.58$) than they did on solution questions ($M = 5.49$, $SD = .53$) ($t(24) = 3.4$, $p < .01$) or on set size questions ($M = 5.84$, $SD = .37$) ($t(24) = 4.48$, $p < .001$) (see Figure 9 and Tables 1). Participants also scored significantly lower on solution questions than they did on set size questions ($t(24) = 2.98$, $p < .01$). This indicates that relationship formation questions are the most difficult, followed by solution questions and finally set size identification questions are the easiest.

Because relationship formation questions were demonstrated to be the most difficult, further investigation was conducted into whether this difficulty was present in high and low scorers, examined separately. Interestingly, the difficulty of relationship questions as compared to the other question types was much more prominent in "low scorers" than in "high scorers." This indicates that the main difference in ability between high scorers and low scorers may be in their relationship formation ability.

Facilitation Effects. The experimental design allowed for within subjects comparisons of performance for different question sequences (see Figure 3). For all participants, facilitative effects of problem type were examined: first of set size identification on relationship formation accuracy, second of relationship formation on problem solving accuracy, and finally of set size identification on problem solving accuracy.

The first was done by comparing the scores on RQ1 to scores on SRQ2. By doing this, we could compare performance on relationship formation questions alone (RQ1) to relationship formation questions which followed a set size identification question (SRQ2). A marginally significant facilitation effect was found ($t(24) = 1.32$, $p = .1$, one tailed) with people performing slightly more accurately on the relationship formation question when presented first with a set size identification question (see Figure 12).

The facilitative effect of relationship formation on problem solving accuracy was explored by comparing the scores on Q to the scores on RQ2, following the same logic as the previous comparison. However, no such facilitation was found. There was however a very slight trend indicating that a facilitation effect may be present had there not been such a large ceiling effect on solution question accuracy. The facilitative effect of set size identification on problem solving was investigated by comparing the scores on RQ2 to scores on SRQ3, following the same logic as the first comparison. No significant facilitation effect of set size identification on problem solving accuracy was found. Again, however, participants scored slightly higher on SRQ3 questions than RQ2 questions indicating a trend that might have demonstrated a significant facilitation had there not been a ceiling effect of solution question accuracy (see Figure 9 and Table 1).

Cognitive Processing Differences Between High and Low Accuracy Problem Solvers

The performance of high and low scorers was compared across all problem types. As was to be expected, due to the definition of “high” and “low” scorers, high scorers performed significantly better than low scorers on every problem type (see Table 3 and Figure 11). Based on observed performance, we wished to investigate the cognitive processing differences between the two groups. Because of the limited amount of time available for this study, these groups were too large to allow for the depth of eye movement analyses desired. Given these time constraints, eye movement patterns were examined for a subset of the participant population.

Initial analyses of the eye movement data revealed significant variability in the amount of time participants spent looking at the visual stimuli as a whole, with some participants spending unrealistic amounts of time looking at the computer screen. Based on examination of box plots, outliers were identified and removed. In the second phase of data analysis, the five "highest scorers" and five "lowest scorers," as measured by accuracy scores on the experimental task were selected in order to conduct an extreme groups comparison of eye movement data. Because the purpose of eye movement analyses was to determine differences in attentional allocation of high and low accuracy math problem solvers, an examination of extreme groups allowed for an exploratory comparison between the participants performing at the upper and lower end of the spectrum. After the removal of outliers, the ten selected scorers consisted of five "highest" scorers ($M = 35.8$, $SD = 0.447$) and five "lowest" scorers ($M = 27.8$, $SD = 2.17$). It is important to note that four out of the five "highest" scorers achieved a perfect overall score of 36. This lack of variability does not allow for improvement across trials or different problem types.

Re-analysis of the facilitation effects revealed that set size identification facilitated relationship formation accuracy for lowest scorers ($t\ (4) = 4.81$, $p < .01$), but not highest scorers. The absence of a facilitation effect for high scorers may again be attributed to the ceiling effect seen in the scores of highest scorers. Additionally, relationship formation marginally facilitated solution accuracy in lowest scorers ($t\ (4) =$

1.63, $p = .08$, one tailed). This may indicate that problem solving accuracy can be increased in low ability problem solvers by providing prompts, such as set size identification and relationship formation.

Eye Movement Analyses. The significant difference in relationship formation ability in high scorers and low scorers was even more prominent between the highest scorers and the lowest scorers. Eye movement data was analyzed in order to assess differences in attention allocation between highest scorers and lowest scorers during relationship formation tasks. Specifically eye movements during the RQ1 question were assessed and compared between groups to identify trends in the allocation of attention to each of five areas of interest: 1. the assignment sentence, 2. the relational sentence, 3. the definition of variables, 4. the relational question, and 5. the multiple choice options (see Figure 7). Eyenal, a data analysis program used with the ASL 501 eye tracking system, provided information on total fixations within each area of interest and mean fixation duration within each area of interest; from that, the total amount of time, on average, each participant spent fixating within a specified area of interest was computed. Another program, called FixPlot allowed for the overlay of eye movement patterns on top of specified visual fields (see Figure 13). Although this data could not be directly analyzed, it provides insight into the nature of the visual scan paths.

Analysis revealed that there were no significant differences between highest scorers and lowest scorers in terms of fixation duration, number of fixations or total time spent in each of the five areas of interest as measured by multiple t-tests. However, trends were present in the data, suggesting differences in the attentional allocation of highest and lowest scorers. Because of the nature of this exploratory analysis, the sample examined was extremely small; therefore, it was hypothesized that the lack of significance as demonstrated by t-tests might have been due to the limited sample size. In order to test for an effect without being limited by the small sample size, effect sizes were used to examine the differences between groups. An effect size, *d*, represents the size of an effect by examining the difference between the means of the groups in question and comparing this difference to the average of the standard deviations of the two groups. In this way, a difference can be converted into standard deviation units:

$$d = \text{mean } 1 - \text{mean } 2/\text{SD},$$

where SD represents the average of the standard deviations of the two groups (Rosenthal & Rosnow, 1991). Absolute values of *d* that correspond to small, medium and large effects are .20, .50 and .80 respectively (Rosenthal & Rosnow, 1991 p. 444).

Effect sizes were computed to identify potentially informative differences between high and low scorers in terms of attentional allocation as measured by eye movement patterns (see Figure 14, and Tables 6 and 7). It was revealed that highest scorers spent a larger percent of their time fixating on the definition of variables ($d = .856$) and on the question ($d = .704$) than lowest scorers, as measured by the percent of total time spent looking within each area. Additionally, highest scorers demonstrated a greater number of fixations within the question than did lowest scorers ($d = .824$), as measured by the total number of fixations within each area. This could indicate the importance of attending to the question and understanding what the question is asking in problem solving.

Interestingly, lowest scorers exhibited a greater number of fixations within the multiple choice options ($d = -1.01$) and within the relational statement ($d = -.751$) than did highest scorers, as measured by the total number of fixations within each area. This could indicate an active comparison between potential answers and initial problem information, and not enough attention to the question being asked.

Discussion

The importance of mathematical relationship formation in word problem solving was examined in order to detect trends in the problem solving process. Five main hypotheses were investigated. These hypotheses were: 1.) mathematical relationship formation is more difficult for the average problem solver than set size identification; 2.) mathematical relationship formation ability is positively correlated with solution accuracy; 3.) mathematical relationship formation prompts will facilitate subsequent solution accuracy; 4.) set size identification prompts will facilitate subsequent relationship formation accuracy; and 5.) high accuracy problem solvers will spend more time fixating relational statements than low accuracy problem solvers during the solution process.

Results supported the first hypothesis; relationship formation questions had the lowest accuracy rates as compared to set size identification questions and solution questions. This supports the proposal that relationship formation is in fact the most difficult part of the problem solving process. It is important to note that the average accuracy score for both the set size identification questions and the solution questions were near perfect, implying a ceiling effect on these accuracy measures. This suggests that, although the relationship formation questions posed a challenge to participants, the set size identification questions and solution questions did not.

Interestingly, on many three-step problems (SRQ problems) some participants who were first able to correctly answer the set size identification question, went on to incorrectly answer the relationship formation question, but then proceeded to correctly answer the solution question. This poses an interesting question: if relationship formation is important to problem solving accuracy, then how can participants arrive at an accurate solution after incorrectly answering a relationship formation question? Due to the ceiling effect seen in solution questions, it is hypothesized that the solution questions were, in fact, too easy for the studied population. Due to their relative simplicity, it is possible that the participants who were not able to form accurate relationships have developed alternative strategies to solving simple word problems that do not involve mathematical relationship formation. According to this theory, if the solution question was so complex that accurate solution depended on the formation of an accurate mathematical relationship, we would expect to see the solution accuracy drop in those with poor relationship formation skills.

To further investigate the importance of relationship formation on problem solving accuracy, the second hypothesis explored a potential association between relationship formation ability and solution accuracy. No significant correlations were found between the relationship formation questions and the solution questions, indicating that there is no meaningful association between the two. However, although an association was not present in the current study, it is reasonable to hypothesize that it may be present in other problem solving situations. As mentioned previously, the low variability of the solution scores, due to a ceiling effect, might have made it difficult to expose meaningful associations. An examination of more complex word problems might uncover associations between relationship formation and problem solving accuracy.

The third hypothesis explored a facilitation effect of relationship formation prompts on solution accuracy. Again, the high accuracy and low variability of solution questions made it difficult to reveal a facilitative effect, as there was little room for improvement between simple solution questions and solution questions following a relationship formation question. In this study, participants performed slightly better on solution questions that followed relationship formation questions than on solution questions presented alone. That is, relationship formation prompts marginally facilitated problem solving accuracy. This finding is quite substantial considering the extreme skew of the solution question scores, and it is hypothesized that this facilitation effect would have been more prominent had the experimental test been more difficult.

Understanding the importance of mathematical relationship formation on problem solving accuracy can have significant implications for education. Since relationship formation ability appears to facilitate problem solving accuracy, this would suggest the importance of teaching students how to create accurate mathematical representations of the text while solving word problems. If educational institutions strive to improve the relationship formation skills of their students, stronger problem solvers may be fostered. But how can these relationship formation skills be improved?

In order to investigate ways in which relationship formation skills can be improved, the fourth hypothesis was addressed: the association between set size determination and relationship formation ability. A substantial facilitation effect of set size identification prompts on relationship formation accuracy was discovered. That is, when prompted to first identify the relative size of the sets in question before attempting to create a mathematical relationship, participants performed significantly better than when they were not given the initial prompt. This finding suggests that relationship formation skill can be improved by encouraging students to think about the relative size of the sets in question prior to attempting to form a mathematical relationship between them.

Taken together, the preceding two results support the proposed cognitive model (see Figure 1). We have identified a facilitative effect of set size identification prompts on relationship formation and of relationship formation prompts on solution accuracy; therefore it is logical to conjecture that the proposed cognitive model may represent an element of the cognitive processes involved in math problem solving. That is, it may be beneficial to first determine the relative size of the sets in the problem, to then use this information to create a mathematical representation of the text, and to finally solve this mathematical representation in order to arrive at a solution. This model breaks problem solving into a series of manageable steps that appear to facilitate solution accuracy. Based on these findings, it might be beneficial for educators to direct their efforts toward teaching the skills necessary to perform accurately in the stages that precede solution acquisition, in order to facilitate accurate problem solving.

How then, can these skills be taught to aspiring problem solvers? Perhaps the ability of effective problem solvers to step through the proposed cognitive model of set size identification, relationship formation, and solution acquisition has to do with the way in which attention is allocated during the problem solving process. This study explored the attentional allocation of high accuracy vs. low accuracy problem solvers, as measured by eye movement analyses. The goal of this portion of the investigation was to determine if, during the steps preceding solution acquisition, high accuracy problem solvers were attending to different parts of the problem than low accuracy problem solvers. Based on the finding that relationship formation ability facilitates solution accuracy, attentional allocation during relationship formation was specifically examined.

Interestingly, results did not support the hypothesis that high accuracy problem solvers would direct more attention to the relational statement than low accuracy problem solvers, but in fact, suggested the opposite. Instead, findings indicate that low accuracy problem solvers attend more to the relational statement than high accuracy problem solvers, and high accuracy problem solvers attend more to the question than low accuracy problem solvers, as measured by eye movement analyses. However, since the relational statement contains the information necessary for relationship formation, it clear that since high accuracy problem solvers are forming accurate mathematical relationships, they must be devoting a sufficient portion of their attention to the relational statement. These results simply indicate that low accuracy problem solvers, are directing their attention to the appropriate place as well, however they may not know what they are looking for in the relational statement or what to do with the information they are attending.

Suppose that both high and low accuracy problem solvers recognize the importance of allocating attention to the relational statement in order to select the appropriate information for relationship formation, but go about the process of information selection in different ways. It is possible that high accuracy problem solvers allocate more attention to the question in order to determine what is required of them, and are then able to rapidly select the necessary information from the relational statement, without devoting much attention to this selection process. According to this theory, it could be the case that low accuracy problem solvers spend more time searching within the relational statement for the appropriate information for relationship formation. This could be because low accuracy problem solvers have not paid enough attention to the question in order to easily identify the information needed to form an accurate relationship. According to this hypothesis, the findings of this study do not suggest that high accuracy problem solvers are inattentive to the relational statement, but instead indicate the importance of the question in guiding information selection and solution formation. This theory supports the findings of Cook & Rieser (2005) who discovered that the most effective method for information selection is "question guided comparison," in which special attention is paid to the question in order to identify what types of information are being sought. In the context of the present study, high accuracy problem solvers may pay more attention to the question in order to quickly identify the appropriate information in the relational statement.

The eye movement analyses conducted in the present study specifically focused on eye movement patterns during relationship formation tasks. Because relationship formation is important to problem solving accuracy, understanding attentional allocation during this process provides insight into the cognitive processes that occur during relationship formation. The identification of specific areas that are more frequently attended by high accuracy problem solvers can allow educators to appropriately direct aspiring problem solvers to attend to these important areas.

Because relationship formation ability can facilitate problem solving accuracy, and set size identification ability can facilitate relationship formation accuracy, it would be interesting to investigate attentional allocation differences between high and low problem solvers during the set size identification task. Future research should examine attentional allocation differences in high and low accuracy problem solvers in each of the three steps in the proposed cognitive model. Results of such studies may allow researchers to fill in the gaps of the problem solving process in order to enhance the proposed cognitive model. Ultimately, this model may be used to shape accurate problem solvers.

Limitations. This research was conducted over a one year period during which the study was designed, conducted and analyzed. Time constraints severely limited the extent to which data could be examined. Eye movement data was collected during the entire experimental task; however only the data associated with relationship formation questions was examined. Future analysis of the collected data should be done in order to identify attentional allocation patterns associated with the different steps in the proposed cognitive model, as measured by eye movement patterns.

Time constraints also made it difficult to properly test the experimental design prior to participant testing. The experimental task was designed to allow for an examination of the different steps in the proposed cognitive model. Unfortunately, pilot tests were not conducted with the testing material to determine whether the questions would appropriately challenge a college population. Consequently, the experimental task was too easy on many of the question types, which was demonstrated by a ceiling effect in the accuracy scores. The lack of variability in scores made comparisons within the data difficult. Future research should ensure that the experimental task is challenging enough to allow for a greater variability of scores.

It is hypothesized that greater facilitation effects of set size identification prompts on relationship formation accuracy and of relationship formation prompts on problem solving accuracy would have been seen if the test problems were more difficult. For example, as discussed earlier, if the solution question was so complex that an accurate solution depended on the formation of an accurate mathematical relationship, we would expect to see solution accuracy drop in those with poor relationship formation skills. This drop in accuracy would leave more room for improvement when relationship formation prompts were presented allowing for further investigation into the importance of relationship formation on problem solving.

There are many factors that contribute to the success of a problem solver, including, but not limited to, computational ability (Mayer, et al. 1991; De Corte, et al. 1990) and reading ability (Cook & Rieser, 2005; Fuchs, Fuchs & Prentice, 2004). This study failed to take into account the computational ability or reading ability of participants. Because a college population was examined, it was assumed that reading ability was at a relatively high level; however the variability of computational ability may be quite high. Future research would benefit from administering a test of computational ability prior to the experimental task in order to allow researchers to separate problem solving skill and computational ability.

Finally, this study was conducted using an eye tracking system that was recently purchased by Hamilton College. A significant amount of time was devoted to the setup of and the familiarization with the new technology. Increased knowledge of and improved skill using the eye tracking system would have been beneficial to the ease and efficiency of the research. Due to these constraints, it is possible that experimental error was present during data collection.

Future Research. This study has investigated relationship formation in math problem solving and uncovered many interesting trends in the cognitive processes involved in both relationship formation and math problem solving as a whole. The facilitation effects that were identified should be examined in more depth in order to determine ways in which these findings can be applied to educational situations. For example, a facilitation effect of set size identification prompts on relationship formation accuracy was found. That is, when participants were presented with a problem such as the one in Figure 6, they were first prompted to determine the larger of two sets: in this case $C < S$. They were then asked to form a relationship between the variables C and S, with the correct solution being $3C = S$. It was demonstrated that participants performed better on this relationship formation task when they were first presented with the set size identification task (as in Figure 6), than when they were not (as in Figure 5). It would be beneficial to understand how this facilitation works. That is, how does identifying $C < S$ help people create the relationship $3C =S$? Future research should be conducted into the ways in which problem solvers convert a statement such as $C < S$ to a statement that continues to show S as the larger set: $3C = S$.

One way to investigate this process would be to examine how it is learned. This study focused on a college participant pool, most of whom are no longer receiving math education. However, the skills that are necessary for solving simple math word problems are learned very early in life. Investigation into how the cognitive processes associated with math problem solving are formed would be quite beneficial to understanding the processes later in life. Additionally, it may provide insight into the difficulties some students have in math problem solving.

This study has closely examined mathematical relationship creation as it applies to math problem solving accuracy. Many theories have been suggested which, if applied to education, may help to improve the math skills of learners. Applying the findings of this study to math education would be one way to further investigate the usefulness of the proposed cognitive model. The creation and implementation of intervention techniques that apply the proposed cognitive model may help to educate youth and improve mathematical achievement and proficiency in our society.

References

Ashcraft, M. (2002). *Cognition* (3rd ed.). Upper Saddle River: Prentice Hall Inc.

Cook, J., & Rieser, J. (2005). Finding the Critical Facts: Children's Visual Scan Patterns When Solving Story Problems That Contain Irrelevant Information. *Journal of Educational Psychology,* 97, 224-234.

De Corte, E., Verschaffel, L., & Pauwels, A., (1990). Influence of the Semantic Structure of Word Problems on Second Graders' Eye Movements. *Journal of Educational Psychology,* 82, 359-365.

Fry, C. (1987*). Eye fixation patterns in the solution of mathematical word problems by young adults: Relation to cognitive style and spatial ability.* Unpublished doctoral dissertation, The Ohio State University, Columbus.

Fuchs, L., Fuchs, D., & Prentice, K. (2004). Responsiveness to Mathematical Problem-Solving Instruction: Comparing students at risk of mathematical disability with and without risk of reading disability. *Journal of Learning Disabilities*, 37, 293-306.

Hegarty, M., Mayer, R., & Green, C. (1992). Comprehension of Arithmetic Word Problems: Evidence From Students' Eye Fixations. *Journal of Educational Psychology,* 84, 76-84.

Hinsley, D., Hayes, J., & Simon, H. (1977). From Words to Equations Meaning and Representation in Algebra Word Problems. In M. Just, & P. Carpenter (Eds.), *Cognitive Processes in Comprehension* (pp. 89-106). Hillsdale, New Jersey: Lawrence Erlbaum Associates, Inc., Publishers.

Kintsch, W., & Greeno, J. (1985). Understanding and Solving Word Arithmetic

Problems. *Psychological Review*, 92, 109-129.

Lewis, A., & Mayer, R. (1987). Students' Miscomprehension of Relational Statements in Arithmetic Word Problems. *Journal of Educational Psychology,* 79, 363-371.

Mayer, R., Tajika, H., & Stanley, C. (1991). Mathematical Problem Solving in Japan and the United States: A Controlled Comparison. *Journal of Educational Psychology*, 83, 69-72.

Michie, S. (1985). Development of Absolute and Relative Concepts of Number in Preschool Children. *Developmental Psychology*, 21, 247-252.

Moreau, S., & Coquin-Viennot, D. (2003). Comprehension of arithmetic word problems by fifth-grade pupils: Representations and selection of information. *British Journal of Educational Psychology, 73*, 109-121.

Nesher, P. (1986). Learning Mathematics: A Cognitive Perspective. *American Psychologist,* 41 1114-1122.

Osterroth, P. (1994). Variations in problem solving ability and levels of conceptual development. *Journal of Instructional Psychology*, 21, 265-267

Posner, M., Cohen, Y., & Rafal, R. (1982). Neural systems control of spatial orienting. *Phil. Trans. R. Soc. Lond.* B 298, 187-198.

Posner, M., & Cohen, Y. (1984). Components of Visual Orienting. *Attention and Performance X,* 32, 531-556.

Roberge, J., & Flexer, B. (1983). Cognitive Style, Operativity, and Mathematics Achievement. *Journal for Research in Mathematics Education*, 14, 344-353.

Roman, H. (2004). Why math is so important. *Tech Directions*, 63, 16-18.

Rosenthal, R., Rosnow, R. (1991). *Essentials of Behavioral Research: Methods and Data*

Analyses (2nd ed.). Boston: McGraw Hill.

Swan, P. (2004). Computation choices made by children: What, why and how. *Australian Primary Mathematics Classroom*, 9, 27-30.

Tajika, H. (1994). A cognitive component analysis of arithmetic word problem solving. In J. E. H. Van Luit (Ed.), *Research on Learning and Instruction of Mathematics in Kindergarten and Primary School* (pp. 242-250). Doetinchem, The Netherlands: Graviant Publishing Company.

Terry, P. (1992). The Reading Problem in Arithmetic. Journal *of Educational Psychology,* 84 70-75.

Verschaffel, L., De Corte, E., & Pauwels, A. (1992). Solving Compare Problems: An Eye Movement Test of Lewis and Mayer's Consistency Hypothesis. *Journal of Experimental Psychology,* 84, 85-94.

Appendix 1

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Psychology Department

Consent Form

Purpose:
The purpose of this study is to investigate the processes involved in mathematical word problem solving. The study is part of Kalin Jaffe's senior thesis in Psychology, under the supervision of Professor Penny Yee.

Procedure:
If you agree to be in this study, you will be asked to do the following:
1. Wear a headpiece with a camera mounted on it in order to monitor your eye movements.
2. Read a series of stimuli presented to you on a computer screen and answer questions regarding this stimuli.
3. Actively solve math word problems.

The total time required to complete the study should be approximately 30 minutes. You will be compensated for participation in this study with a coupon to Café Opus.

Benefits/Risks to Participant:
Participants will have the opportunity to be a part of some of the first research conducted with the ASL 500 Eye Tracking System at Hamilton College. They will learn about the procedures involved in eye tracking research and will contribute to some of the first mathematical research using the eye tracker. Risks include any discomfort you may feel while wearing the eye-tracking headpiece and performing mathematical tasks.

Voluntary Nature of the Study/Confidentiality:
Your participation in this study is entirely voluntary and you may refuse to complete the study at any point during the experiment. You may also stop at any time and ask the researcher any questions you may have. Your name will never be connected to your results or to your responses; instead, a number will be used for identification purposes. Information that would make it possible to identify you or any other participant will never be included in any sort of report. The data will be accessible only to those working on the project.

Contacts and Questions:
At this time you may ask any questions you may have regarding this study. If you have questions later, you may contact Kalin Jaffe at x2722 or kjaffe@hamilton.edu or Supervising Professor, Penny Yee at x4720 or pyee@hamilton.edu. Questions or concerns about institutional approval should be directed to Penny Yee, Chair of the Institutional Review Board for Human Subjects, x-4720 or pyee@hamilton.edu.

Statement of Consent:
I have read the above information. I have asked any questions I had regarding the experimental procedure and they have been answered to my satisfaction. I consent to participate in this study.

Name of Participant____Date: ___________
(Please print)
Signature of Participant ____

Age: ________**(Note: You must be 18 years of age or older to participate in this study. Let the experimenter know if you are under 18 years old.)**

Thanks for your participation!

Table 1

Overall Accuracy Scores for Each Question Type

Question	Mean Score	Standard Deviation
Q	5.44	.651
RQ1	4.28	1.696
RQ2	5.48	.714
SRQ1	5.84	.374
SRQ2	4.60	1.683
SRQ3	5.56	.768

Note. The values represent mean scores of correctly answered questions, where Q, RQ2 and SRQ3 are solution questions; RQ1 and SRQ2 are relationship formation questions, and SRQ1 is a set size identification question.

Table 2

Overall Response Times for Each Question Type

Question	Mean Response Time	Standard Deviation
Q	28.947	10.539
RQ1	45.862	17.079
RQ2	20.362	15.387
SRQ1	35.990	11.194
SRQ2	26.288	11.284
SRQ3	18.089	6.467

Note. The values represent mean response times for each question measured in seconds, where Q, RQ2 and SRQ3 are solution questions; RQ1 and SRQ2 are relationship formation questions, and SRQ1 is a set size identification question.

Table 3

Accuracy Scores for High vs. Low Scorers on Each Question Type

	High Scorers		Low Scorers	
Question	Mean Score	Standard Deviation	Mean Score	Standard Deviation
Q	5.67	.492	5.23	.725
RQ1	5.58	.669	3.08	1.441
RQ2	5.92	.289	5.08	.760
SRQ1	6.00	.000	5.69	.480
SRQ2	5.67	.492	3.62	1.805
SRQ3	5.83	.389	5.31	.947

Note. The values represent mean scores of correctly answered questions, where Q, RQ2 and SRQ3 are solution questions; RQ1 and SRQ2 are relationship formation questions, and SRQ1 is a set size identification question.

Table 4

Response times for High vs. Low Scorers on Each Question Type

	High Scorers		Low Scorers	
Question	Mean Response Time	Standard Deviation	Mean Response Time	Standard Deviation
Q	25.197	7.284	32.408	12.104
RQ1	39.722	13.033	51.530	18.847
RQ2	15.108	5.387	25.211	19.845
SRQ1	33.798	9.940	38.013	12.280
SRQ2	22.044	7.629	30.206	12.910
SRQ3	17.170	6.560	18.938	6.524

Note. The values represent mean response times for each question measured in seconds, where Q, RQ2 and SRQ3 are solution questions; RQ1 and SRQ2 are relationship formation questions, and SRQ1 is a set size identification question.

Table 5

Accuracy Correlations for All Participants for Each Question Type

	Test Score	Q	RQ1	RQ2	SRQ1	SRQ2	SRQ3
Test Score	—	.337	.881**	.452*	.305	.815**	.552**
Q	—	—	-.041	.513**	-.041	.091	.320
RQ1	—	—	—	.263	.336	.741**	.332
RQ2	—	—	—	—	.143	.062	.173
SRQ1	—	—	—	—	—	-.040	.325
SRQ2	—	—	—	—	—	—	.245
SRQ3	—	—	—	—	—	—	—

Note. **denotes a correlation significant at the .01 level; *denotes a correlation significant at the .05 level

Table 6

Effect Sizes Comparing Percent of Total Time Spent on Each Area Between High and Low Scorers on Relationship Formation Questions

	High Scorers		Low Scorers			
Area of Interest	Mean	Standard Deviation	Mean	Standard Deviation	Mean Difference	Effect Size
Assignment	18.06	10.62	22.42	7.99	-4.36	-0.464
Relation	20.36	11.05	24.85	6.45	-4.49	-0.496
Definition	22.59	13.51	14.06	4.03	8.53	0.856
Question	20.25	13.19	13.23	3.01	7.02	0.704
MC	18.73	9.86	25.43	12.12	-6.71	-0.607

Note. Values of .20, .50 and .80 correspond to small, medium and large effect sizes respectively. Negative effect sizes represent areas in which low accuracy problem solvers attended more than high accuracy problem solvers; positive effect sizes represent areas in which high accuracy problem solvers attended more than low accuracy problem solvers. Percentages represent the percent of time spent looking at each area of interest relative to the rest of the text, where Assignment = Assignment Sentence, Relation = Relational Statement, Definition = Definition of Variables, Question = Relationship formation Question, and MC = Multiple Choice Options for Relationship Formation Question

Table 7

Effect Sizes Comparing Total Number of Fixations in Area Between High and Low Scorers on Relationship Formation Questions

	High Scorers		Low Scorers			
Area of Interest	Mean	Standard Deviation	Mean	Standard Deviation	Mean Difference	Effect Size
Assignment	14.73	11.40	19.66	9.29	-4.93	-0.474
Relation	17.93	8.26	25.17	10.82	-7.23	-0.751
Definition	18.00	8.29	15.37	7.29	2.63	0.337
Question	14.93	5.69	11.36	2.25	3.56	0.824
MC	12.90	5.02	18.17	5.39	-5.27	-1.01

Note. Values of .20, .50 and .80 correspond to small, medium and large effect sizes respectively. Negative effect sizes represent areas in which low accuracy problem solvers attended more than high accuracy problem solvers; positive effect sizes represent areas in which high accuracy problem solvers attended more than low accuracy problem solvers. Scores represent the average number of fixations within each area of interest, where Assignment = Assignment Sentence, Relation = Relational Statement, Definition = Definition of Variables, Question = Relationship formation Question, and MC = Multiple Choice Options for Relationship Formation Question

Figure Captions

Figure 1

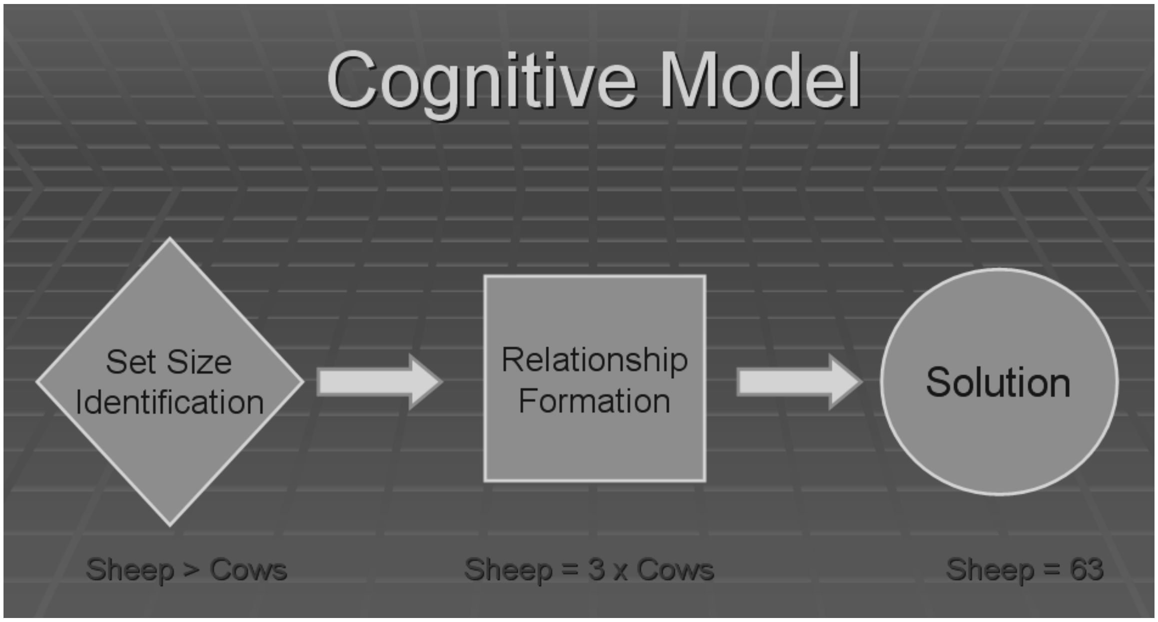

Cognitive Model
Set Size Identification
Relationship Formation
Solution
Sheep > Cows
Sheep = 3 x Cows
Sheep = 63

Figure 2

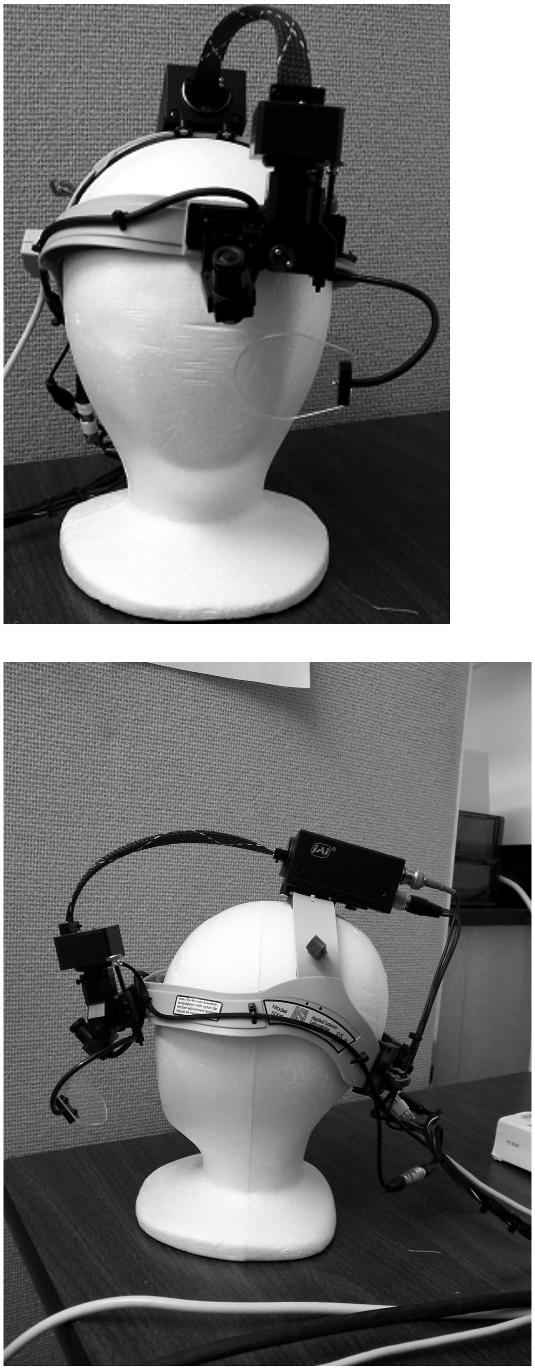


Figure 3

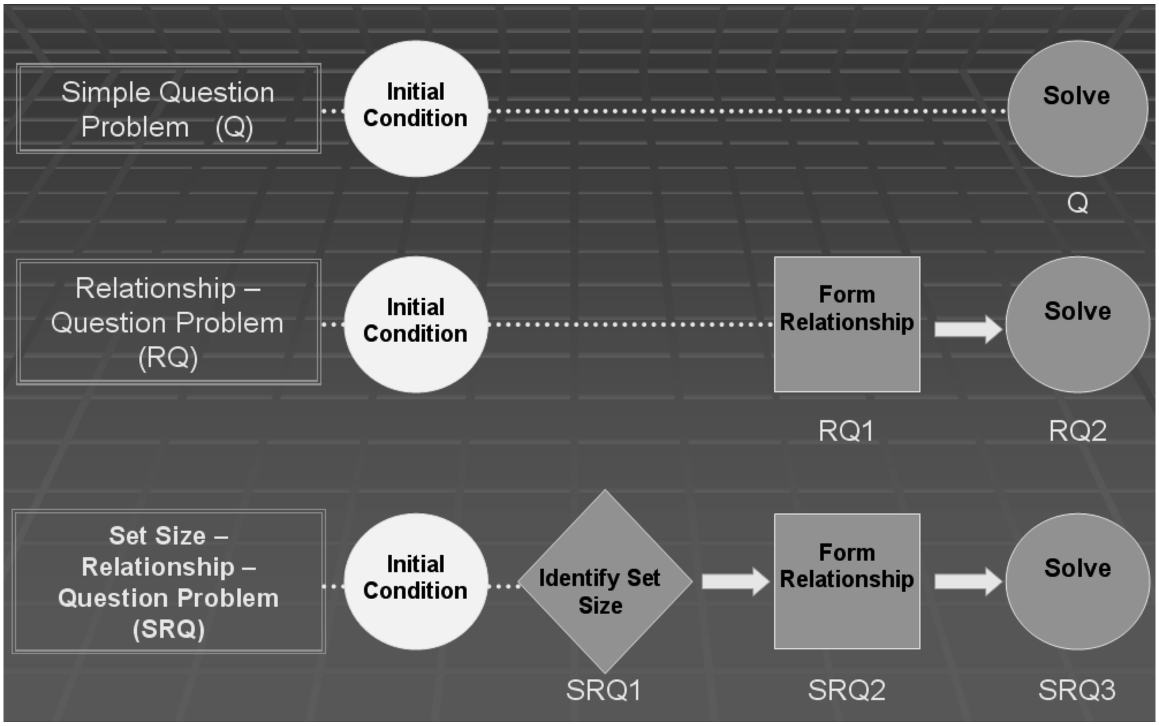

Simple Question Problem (Q)
Initial Condition
Solve
Q
Relationship – Question Problem (RQ)
Initial Condition
Form Relationship
Solve
RQ1
RQ2
Set Size – Relationship – Question Problem (SRQ)
Initial Condition
Identify Set Size
Form Relationship
Solve
SRQ1
SRQ2
SRQ3

Figure 4

On the farm, there are 21 cows. There are three times as many sheep as cows.

How many sheep are on the farm?

Figure 5

On the farm, there are 21 cows. There are three times as many sheep as cows.

Let C represent the number of cows and S represent the number of sheep.

a.) Which equation best expresses the relationship between the number of cows and the number of sheep on the farm?

a. C = 3S
b. 3C = S
c. 21C = S
d. C = 21S

b.) How many sheep are on the farm?

Figure 6

On the farm, there are 21 cows. There are three times as many sheep as cows.

Let C represent the number of cows and S represent the number of sheep.

a.) Which statement best represents the relationship between the number of cows and the number of sheep on the farm?

a. $C > S$
b. $C < S$
c. $C = S$
d. None of the above

b.) Which equation best expresses the relationship between the number of cows and the number of sheep on the farm?

a. $C = 3S$
b. $3C = S$
c. $21C = S$
d. $C = 21S$

c.) How many sheep are on the farm?

Figure 7

P01 the farm, there are 21 cows. There are three times as many sheep as cows.

Let C represent the number of cows and S represent the number of sheep.

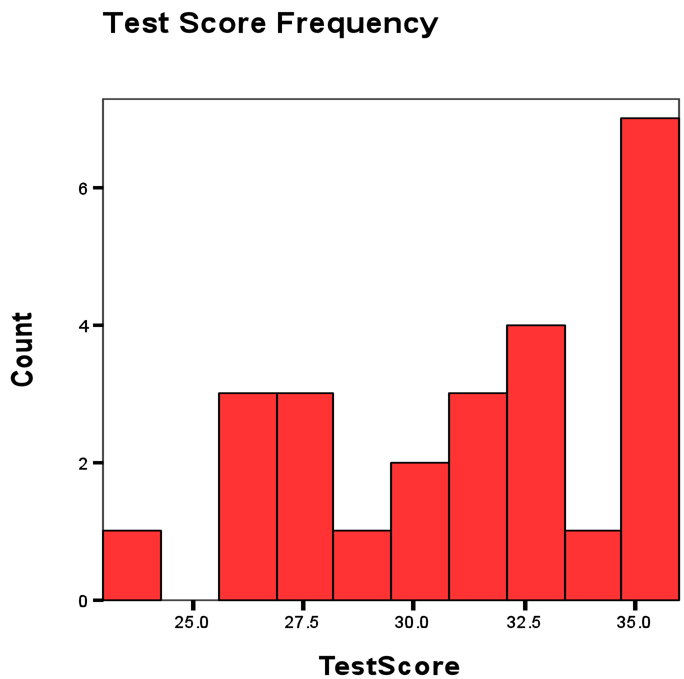
a.) Which equation best expresses the relationship between the number of cows and the number of sheep on the farm?

a. C = 3S
b. 3C = S
c. 21C = S
d. C = 21S

b.) How many sheep are on the farm?

P02

Figure 8



Maximum Score = 36

Figure 9

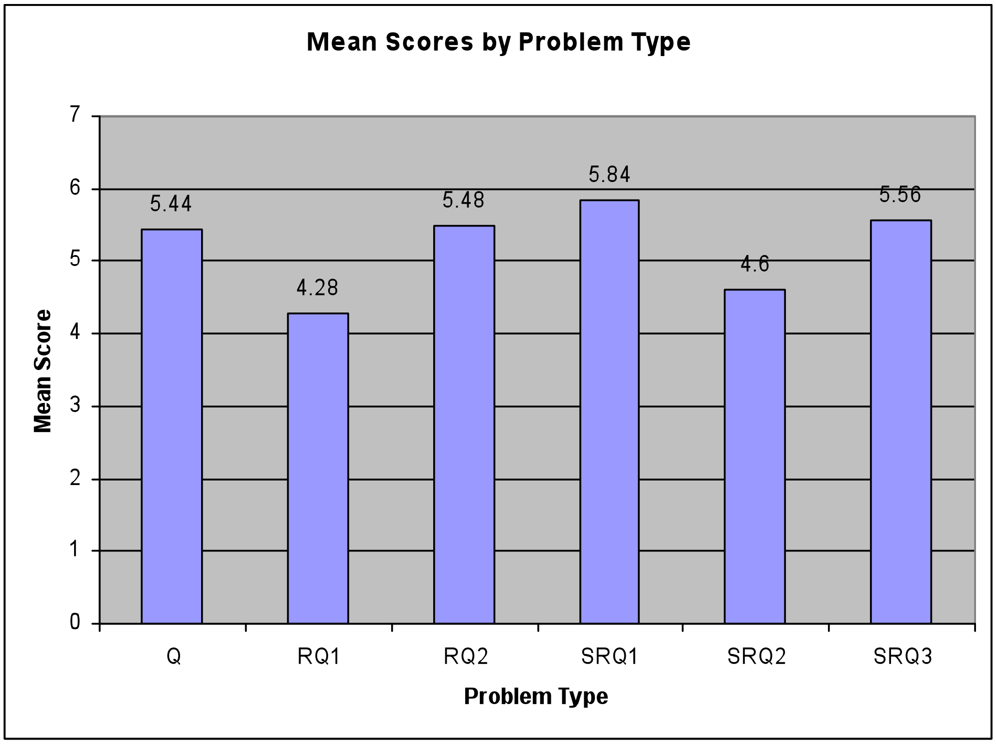

Mean Scores by Problem Type
Mean Score
Problem Type
0
1
2
3
4
5
6
7
5.44
4.28
5.48
5.84
4.6
5.56
Q
RQ1
RQ2
SRQ1
SRQ2
SRQ3

Figure 10

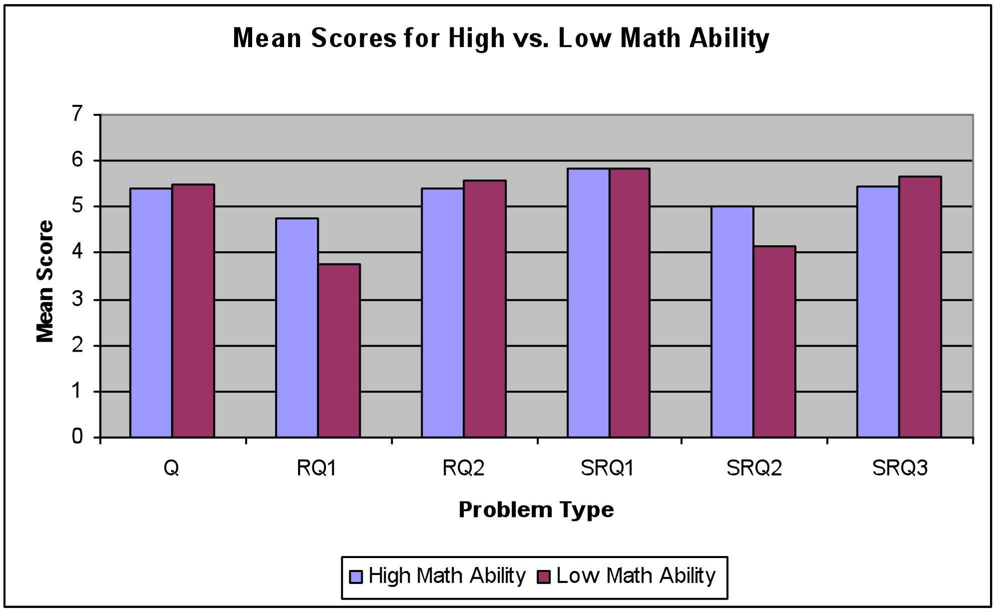

Mean Scores for High vs. Low Math Ability
Mean Score
0
1
2
3
4
5
6
7
Q
RQ1
RQ2
SRQ1
SRQ2
SRQ3
Problem Type
High Math Ability
Low Math Ability

Figure 11

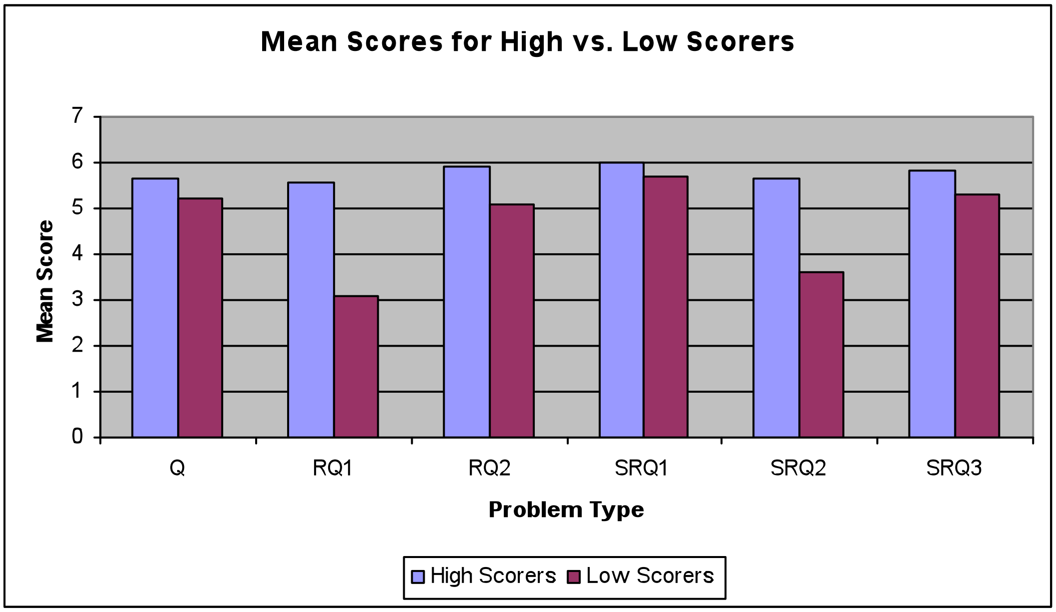

Mean Scores for High vs. Low Scorers
Mean Score
0
1
2
3
4
5
6
7
Q
RQ1
RQ2
SRQ1
SRQ2
SRQ3
Problem Type
High Scorers
Low Scorers

Figure 12

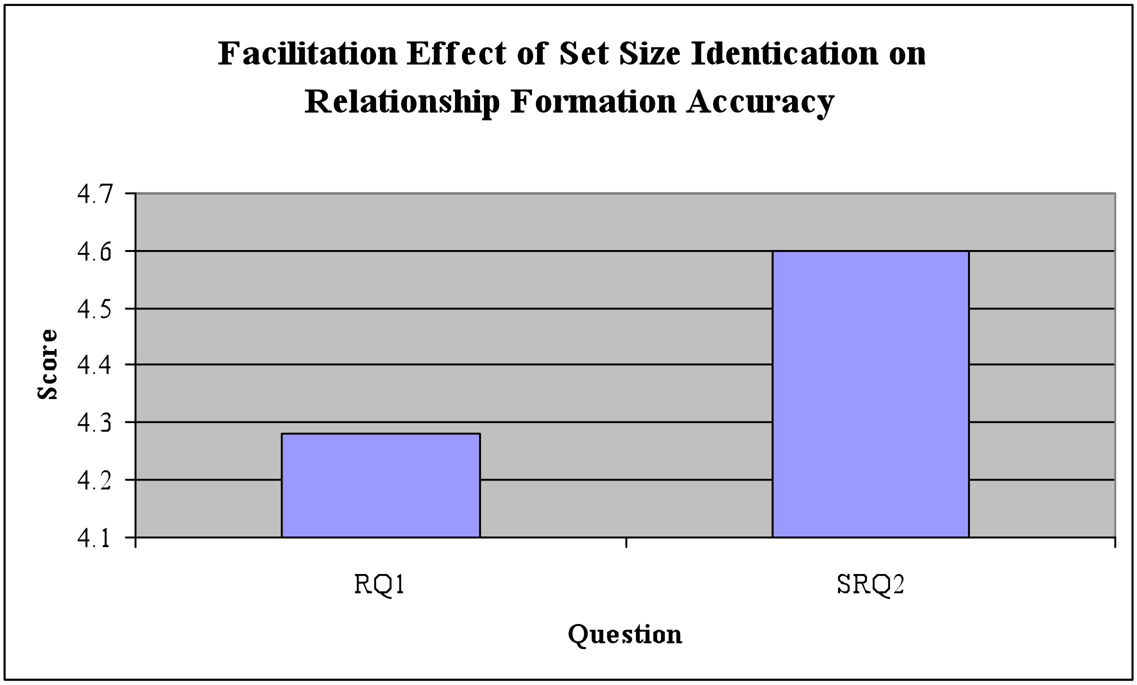

Facilitation Effect of Set Size Identication on Relationship Formation Accuracy
4.7
4.6
4.5
4.4
4.3
4.2
4.1
Score
RQ1
SRQ2
Question

Figure 13

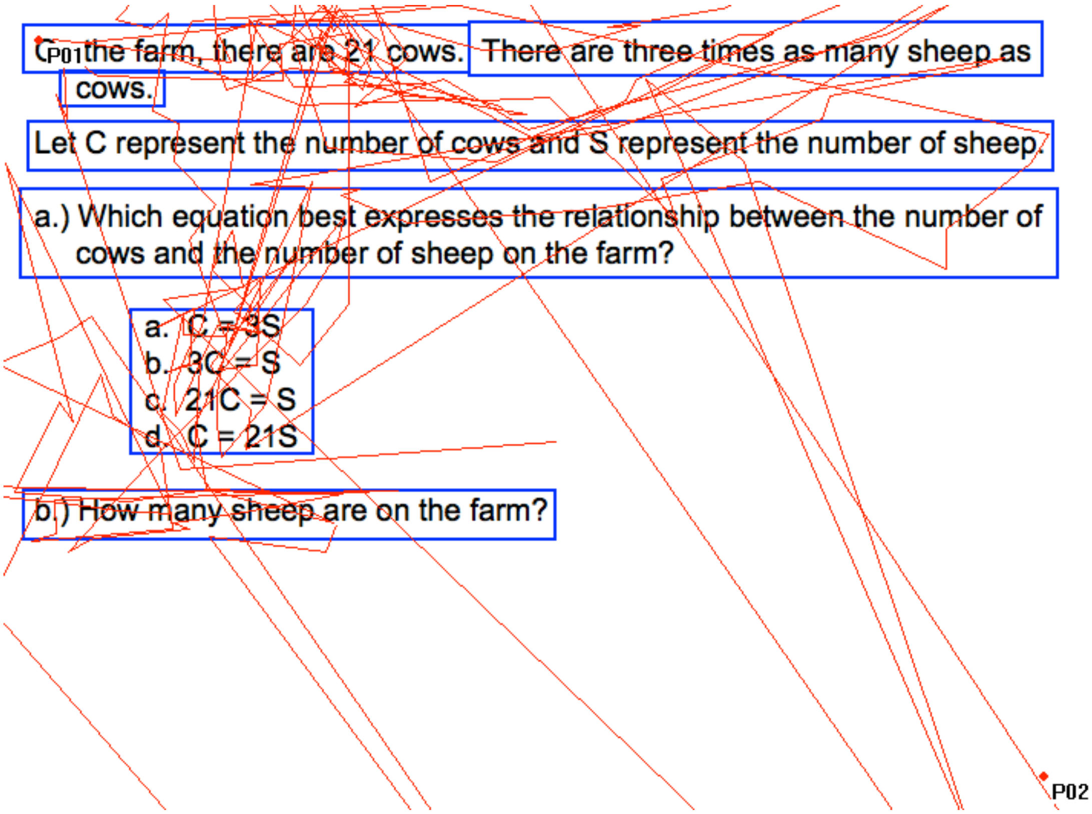
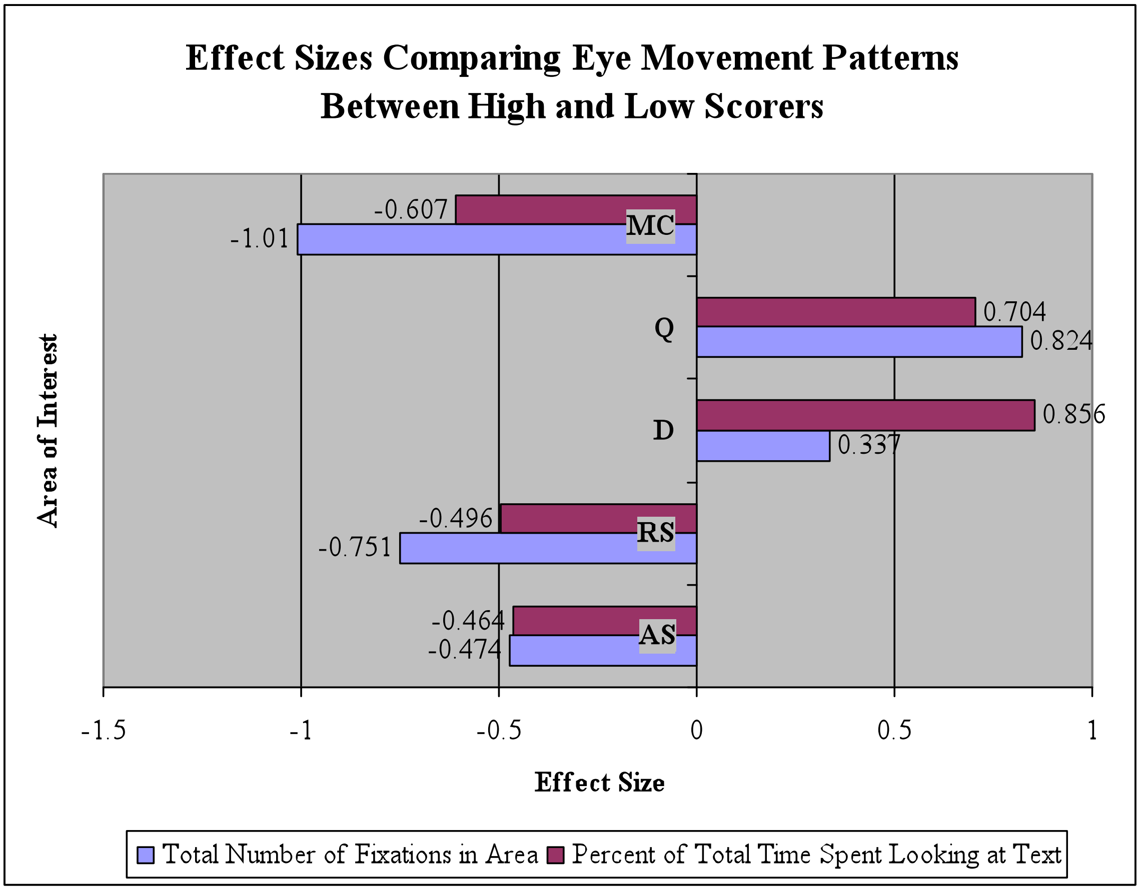

P01
the farm, there are 21 cows.
There are three times as many sheep as cows.
Let C represent the number of cows and S represent the number of sheep.
a.) Which equation best expresses the relationship between the number of cows and the number of sheep on the farm?
a. C = 3S
b. 3C = S
c. 21C = S
d. C = 21S
b.) How many sheep are on the farm?
P02

Figure 14



Note. Negative effect sizes represent areas in which low accuracy problem solvers attended more than high accuracy problem solvers; positive effect sizes represent areas in which high accuracy problem solvers attended more than low accuracy problem solvers, where AS = Assignment Sentence, RS = Relational Statement, D = Definition of Variables, Q = Question, MC = Multiple Choice